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Math 614

Lab 1 – Beetle Census

Dynamic Growth of *Tribolium* Beetle Population Growth

Statement of Problem:

The *Tribolium* flour beetle is a species that causes much damage to flour processing plants. The effects of artificially increasing adult mortality rates creates a dynamical system that can be modeled by the following equations:

$$L_{t+1} = bA_t \exp(-c_{ea}A_t - c_{el}L_t)$$

$$P_{t+1} = L_t(1 - \mu_l)$$

$$A_{t+1} = P_t\,(1 - \mu_p)\exp\big(-c_{pa}A_t\big) + A_t(1 - \mu_a)$$

Where *b* is the birth rate of the species, and where $\mu_l, \mu_p, and\ \mu_a$ are the death rates of the larva, pupa, and adult respectively. The numbers of larvae, pupae, and adults at any given time *t* are denoted by L_t, P_t, and A_t respectively.

The parameters $c_{el} = 0.012$, $c_{ea} = 0.009$, $c_{pa} = 0.004$, $\mu_a = 0.0036$ $\mu_l = 0.267$, $\mu_p = 0$, and $b = 7.48$ were determined from population experiments.

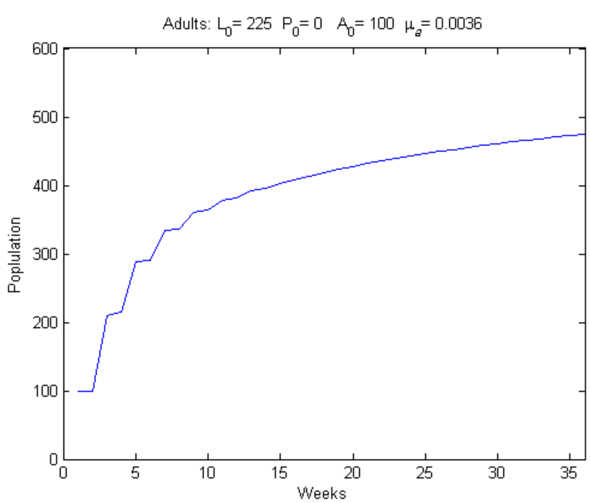
The experiment described in the text began with several hundred beetles. Our goal is to model the data plots from this experiment using MatLab and the growth model above.

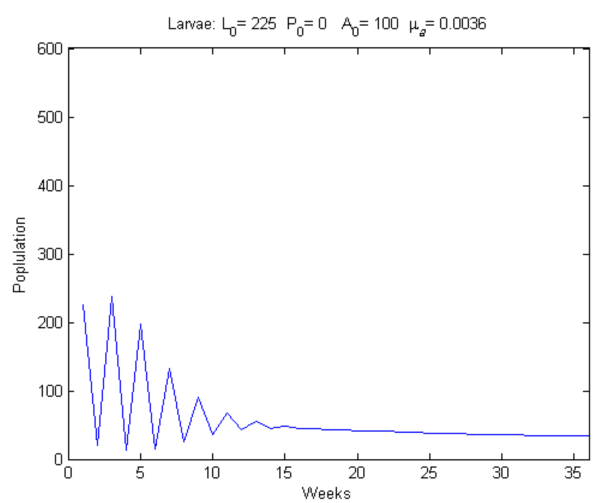
Writeup:

Based off of the graphs provided in the text, I have chosen to initiate the model with $L_0 = 175$, $P_0 = 0$, and $A_0 = 100$ for all the cases below. The adult death rate is changed to model artificial modification of the death rate due to extermination. Using beetle.m (see appendix 1), each model was run for 36 iterations to model 36 weeks.

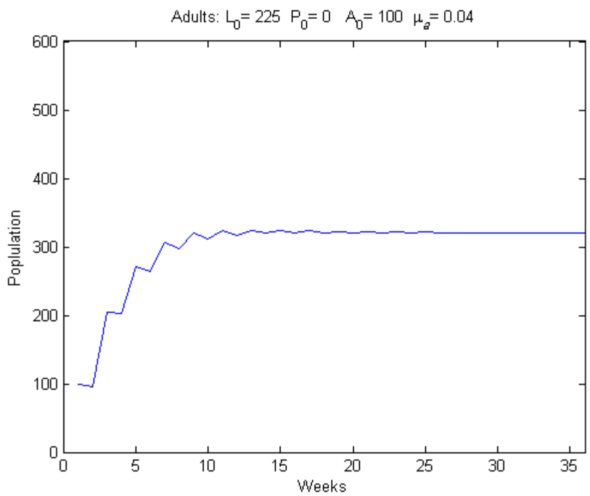
Case 1: Control $\hat{\mu}_a = 0.0036$

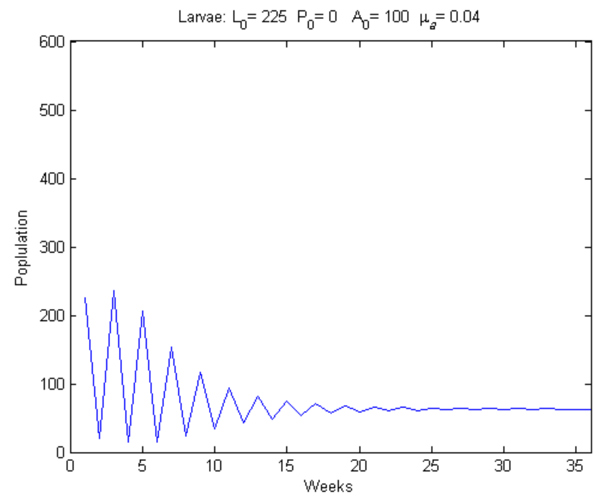
The population of the adults and Larvae fluctuate for the first 10 weeks, but then find a stable equilibrium.



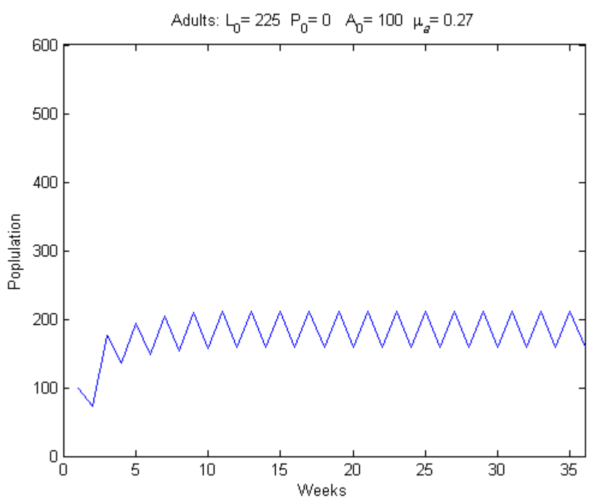


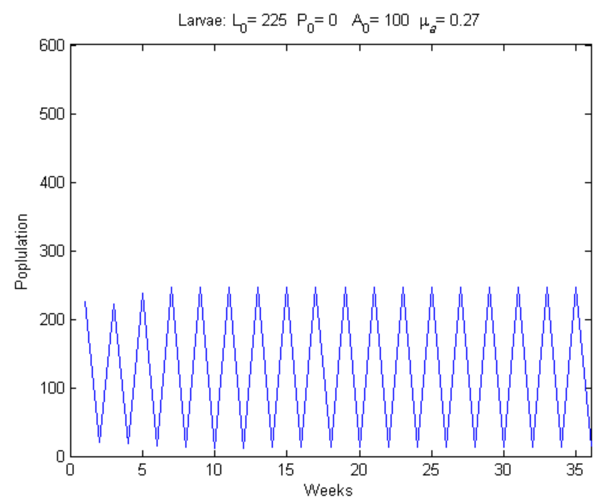
Case 2: $\mu_a = 0.04$ Again the populations of adults and larvae find a stable equilibrium around the 15th week. The total larvae population is similar to case 1, but the adult population is significantly less.



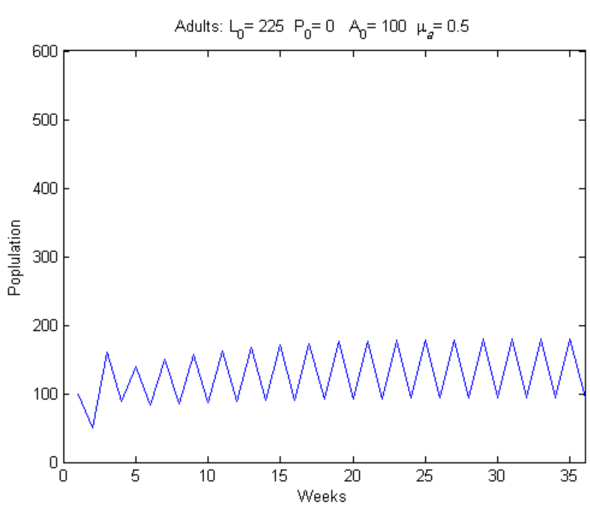


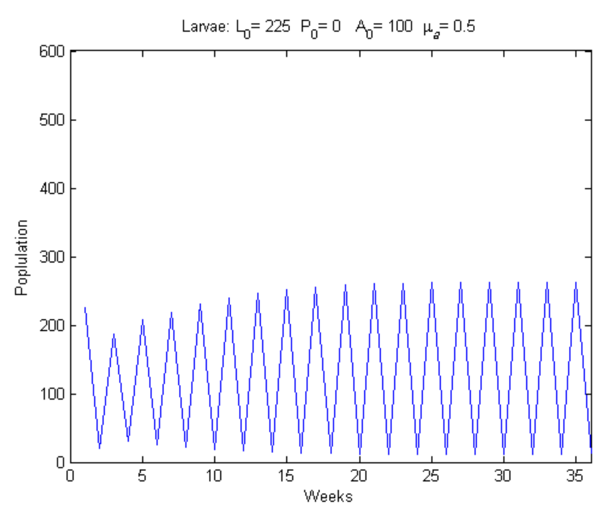
Case 3: $\mu_a = 0.27$ In this case, the population reaches a very clear, stable two-cycle orbit.



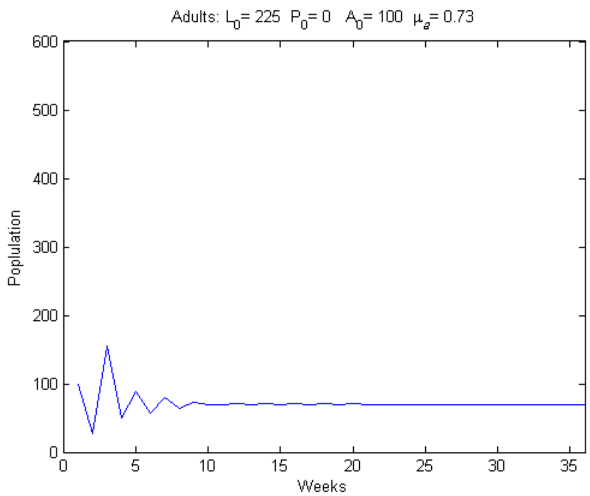


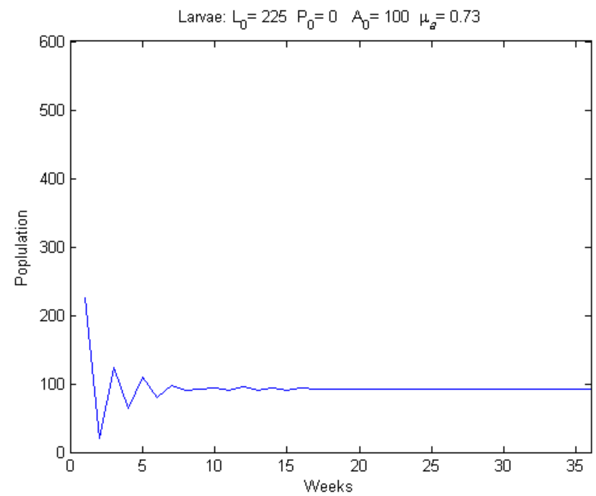
Case 4: $\mu_a = 0.5$ As with case 3, this case reaches a stable two-cycle orbit with a lower average adult population than case 3.



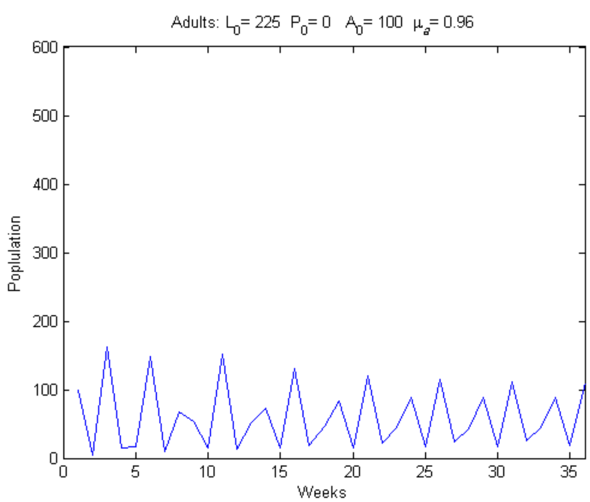


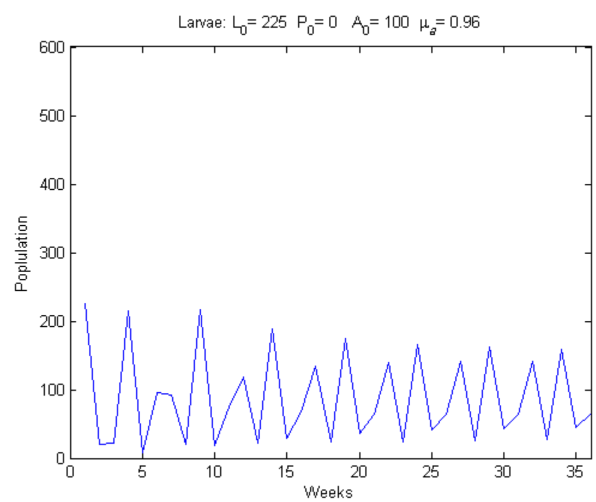
Case 5: $\mu_a = 0.73$ In this case, the model has once again reached an equilibrium at a much lower population than the control case 1. This scenario shows a "best case" that stops the population growth.





Case 6: $\mu_a = 0.96$ Once the adult death rate approaches 1, stability is lost. The plots below show an aperiodic around the stable equilibria in case 5.





References:

1. Chaos: An Introduction to Dynamical Systems, by K. Alligood, T. Sauer, J. A. Yorke. [Springer-Verlag]
2. Linear Beetle Example, Dr. Pilant: http://www.math.tamu.edu/~mpilant/math614/Matlab/linear_beetle.m

Apendix I:

```
function beetle(L0,P0,A0,mua,N)
%Equations 1.6 on page 40 of text.
%Input arguements L0, P0, A0, and N are initial conditions for
%Larvae, Pupae, and Adults, respectively
%as well as mua=adult mortality rate and N=time in weeks

%Initialize static parameters as defined for model 1.6
b=7.48;
cel=0.012;
cea=0.009;
cpa=0.004;
mul=0.267;
mup=0;

%Create and initialize matrices
L=zeros(N,1);
P=zeros(N,1);
A=zeros(N,1);

%Set initial Conditions
L(1)=L0;
P(1)=P0;
A(1)=A0;

%Iterate model N times
for i=1:(N-1)
    L(i+1)=b*A(i)*exp(-cea*A(i)-cel*L(i));
    P(i+1)=L(i)*(1-mul);
    A(i+1)=P(i)*(1-mup)*exp(-cpa*A(i))+A(i)*(1-mua);
end

%Plot Adult variables
t=1:N;
plot(t,A(t));
axis ([0 36 0 600]);

%Apply Lables and Title
XLABEL('Weeks');
YLABEL('Poplulation');
title (['Adults: ','L_0= ', num2str(L0),'  P_0= ',num2str(P0),'   A_0= '  num2str(A0),'
{\mu}_{\ita}= ', num2str(mua)]);

pause

%Plot Adult variables
t=1:N;
plot(t,L(t));
axis ([0 36 0 600]);

%Apply Lables and Title
XLABEL('Weeks');
YLABEL('Poplulation');
title (['Larvae: ','L_0= ', num2str(L0),'  P_0= ',num2str(P0),'   A_0= '  num2str(A0),'
{\mu}_{\ita}= ', num2str(mua)]);
```