Introduction to Intervals

When we are solving equations, we usually have a finite number of solutions. However, when we are solving inequalities, we get a set of numbers that often contain an infinite amount of numbers. These sets of numbers are called intervals. In this lesson, we will develop methods to describe intervals.

Sets of Numbers

- **Natural Numbers:** 1, 2, 3, 4, …
- **Whole Numbers:** 0, 1, 2, 3, 4, …
- **Integers:** …-3, -2, -1, 0, 1, 2, 3, …
- **Rational Numbers:** Can be written as a *ratio* of two integers. Terminating or repeating decimals.

  e.g. $\frac{1}{2}$, $\frac{2}{3}$, .5 , $.333333333333\overline{3} = \frac{1}{3}$, $.125125125\overline{125} = \frac{125}{999}$
- **Irrational Numbers:** Not rational. Non-terminating, non-repeating decimals.

  e.g. $\pi \approx 3.14159\ldots$, $e \approx 2.171828\ldots$, $\sqrt{2}$, $\sqrt[3]{6}$, $etc.$
- **Real Numbers:** The set of all numbers that are *either rational or irrational*.

Describing Sets of Real Numbers

When we want to describe a set of real numbers, it is usually not possible to list all the numbers. So, we will use two types of notation to describe a set.

Set Builder Notation – Uses set brackets { and } to contain the description of the set using inequalities

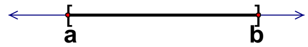
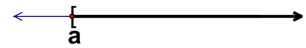
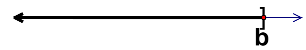
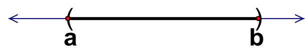
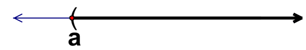
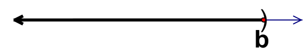
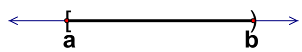
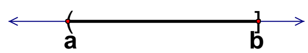
Interval Notation – Shows the *endpoints* of the interval using hard brackets [,] if the endpoint *is included* or soft brackets (,) if the endpoint *is not included*.

Example 1 Describe the set of numbers using a graph, set builder notation, and interval notation.

Inequality	Graph	Set Builder Notation	Interval Notation
$x > 3$			
$x \le -2$			
$2 < x \;\; and \;\; x \le 10$			
$x \le -3 \;\; or \;\; x > 5$			

Summary of Interval Types

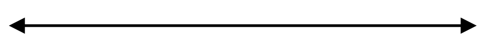
The following is a table that summarizes the possible types of intervals shown in interval notation, as an inequality, and as a graph.

	Bounded Interval	Unbounded Interval	
Closed	$[a, b]$ $a \leq x \leq b$ 	$[a, \infty)$ $x \geq a$  $(-\infty, b]$ $x \leq b$ 	
Open	(a, b) $a < x < b$ 	(a, ∞) $x > a$  $(-\infty, b)$ $x < b$ 	
Half-Open	$[a, b)$ $a \leq x < b$  $(a, b]$ $a < x \leq b$ 	N/A	N/A

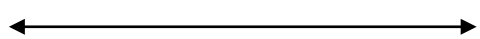
Explore

Represent each set of numbers using a graph, inequality, and interval notation

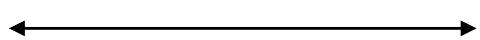
1. The set of all values if x less than or equal to 6.



2. The set of all numbers such that $\sqrt{x}$ is a real number.



3. The set of all numbers such that $x^2 - 4$ is *positive.*



Domains and Interval Notation

The **domain** of a function is the set of all possible inputs (x-values) for a function.

Example 2: State the domain of these **rational functions** in interval notation.

Key Point: *The denominator of a rational function cannot be zero.*

a) $y = \frac{1}{x}$

b) $f(x) = \frac{3x-2}{2x-1}$

Example 3: State the domain of these **radical functions** in interval notation.

Key Point: *The value under the radical in a square-root cannot be negative.*

a) $g(x) = \sqrt{x}$

b) $h(x) = \sqrt{3x+9}$

Example 4: Suppose we have two functions $f(x) = \sqrt{x-2}$ and $g(x) = \sqrt{3x+9}$, find the domain of the following functions:

a) $f + g$

b) $f - g$

Key Point: *The domain of the sum, the difference, or the product of $f(x)$ and $g(x)$ is the intersection of the domains of $f(x)$ and $g(x)$*

Compound Inequalities

When we combine two inequalities with an "and" or an "or", we get a **compound inequality**.

<u>Conjunction</u>: A conjunction is a sentence formed when two or more statements are joined by the word "*and*". In mathematics, the word "and" corresponds to the symbol " $\cap$ ".

The solution set of a conjunction is the intersection of the solution sets of the two statements.

<u>Example 5:</u> Use a graph, set builder notation, and interval notation to describe the set given by the conjunction

$$-1 < x \text{ and } x < 2$$

<u>Example 6:</u> Solve and graph the following conjunctions algebraically. (see video link below)

a) $4 \leq 3x - 5 \ \ and \ \ 2x + 1 < 11$

<u>Video Examples</u>

b) $4 \leq 3x - 5 < 11$

<u>Disjunction</u>: A *disjunction* is sentence formed by joining two or more statements with the word "or". In mathematics, the word "or" corresponds to the symbol " $\cup$ " and the concept of a "union".

<u>Example 7:</u> Solve and graph the following disjunction algebraically. (see video link above)

$$5 - 2x > 17 \quad or \quad 10 \leq 3x - 14$$