Final Exam Review

For each of the following pair of functions, find $f(g(x))$ and simplify.

1. $f(x) = 3x + 4$
   $g(x) = \frac{x}{3} - 4$

   $\mathbf{f(g(x)) = x - 8}$

2. $f(x) = 3x^2 - 5$
   $g(x) = 2x + 1$

   $f(g(x)) = 3(2x+1)^2 - 5$
   $= \mathbf{12x^2 + 12x + 2}$

3. $f(x) = \frac{3x+5}{4}$
   $g(x) = 2x^2 - 3$

   $f(g(x)) = 3(2x^2 - 3) + 5$
   $= \dfrac{\mathbf{3x^2 - 2}}{\mathbf{2}}$

4. $f(x) = 2x + 4$
   $g(x) = \frac{x-4}{2}$

   $f(g(x)) = 2\left(\dfrac{x-4}{2}\right) + 4$
   $= \mathbf{x}$

5. Which of the pairs of functions above are inverses of each other? How do you know?

   The functions in #4 are inverses because $(f \circ g)(x) = x$

6. Simplify. Give the answer in radical notation.

   a. $\sqrt{20x^4y^5}$

   $\mathbf{2x^2y^2\sqrt{5y}}$

   b. $\sqrt[3]{40a^4b^9c^{13}}$

   $\mathbf{2ab^3c^4\sqrt[3]{5ac}}$

   c. $\sqrt[5]{8x^6} \cdot \sqrt[5]{8x^4y^7}$

   $\mathbf{2x^2y\,\sqrt[5]{y^2z}}$

7. Use the properties of exponents and radicals to *simplify*. Write the result using positive exponents.

   a. $\left(x^{-2/5}\right)^{5/4}$

   $= x^{-\frac{2}{5}\cdot\frac{5}{4}} = x^{-\frac{1}{2}} = \dfrac{\mathbf{1}}{\mathbf{x^{1/2}}}$

   b. $\left(\sqrt[6]{a^6b^7}\right)^{12}$

   $= (a^6b^7)^{12/6} = \mathbf{a^{12}b^{14}}$

   c. $\left(\dfrac{27x^4y^7z^{-14}}{xz^{-2}y^4}\right)^{\frac{1}{3}}$

   $= (27x^3y^3z^{-12})^{\frac{1}{3}}$
   $= \dfrac{\mathbf{3xy}}{\mathbf{z^4}}$

   d. $a^{3/8} \cdot a^{2/3}$

   $= a^{\frac{3}{8}+\frac{2}{3}} = \mathbf{a^{\frac{25}{24}}}$

   e. $\dfrac{\sqrt[3]{a^2b}}{\sqrt[6]{ab^3}}$

   $= \dfrac{a^{\frac{2}{3}}b^{\frac{1}{3}}}{a^{\frac{1}{6}}b^{\frac{3}{6}}} = a^{\frac{2}{3}-\frac{1}{6}}b^{\frac{1}{3}-\frac{3}{6}} = a^{\frac{3}{6}}b^{-\frac{1}{6}}$

   $= \mathbf{a^{\frac{1}{2}}} \Big/ \mathbf{b^{\frac{1}{6}}}$

8. Solve the radical equations algebraically. Leave answers in simplest radical form.

a.
$$\sqrt{4-x}+3=5$$
$$\sqrt{4-x}=2$$
$$4-x=4$$
$$\mathbf{0=x}$$

b.
$$\sqrt{x+3}=1+\sqrt{x+2}$$
$$\left(\sqrt{x+3}\right)^2=\left(1+\sqrt{x+2}\right)^2$$
$$x+3=1+2\sqrt{x+2}+(x+2)$$
$$0=2\sqrt{x+2}$$
$$0=\sqrt{x+2}$$
$$0=x+2$$
$$\mathbf{-2=x}$$

c.
$$x+1=\sqrt{2x+3}$$
$$(x+1)^2=\left(\sqrt{2x+3}\right)^2$$
$$x^2+2x+1=2x+3$$
$$x^2=2$$
$$\mathbf{x=\pm\sqrt{2}}$$

Test these answers and...
Only $\sqrt{2}$ works.

9. For each of the quadratic functions, find the vertex, axis of symmetry, the x and y intercepts, domain, range, and graph the function. *Show all your work by hand.*

a. $f(x)=x^2-4x+3$

Axis of symmetry: $x=-\frac{b}{2a}=\frac{-(-4)}{2(1)}=2$

Vertex: $\mathbf{(2,-1)}$

x-intercept(s): $0=x^2-4x+3$

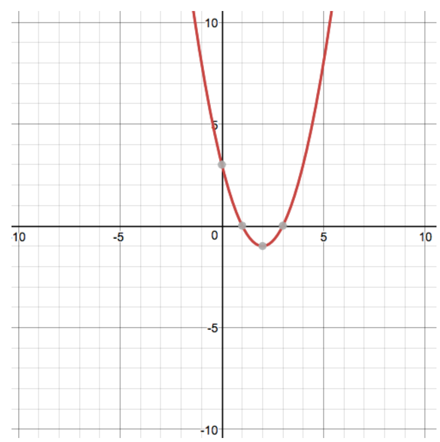
(1,0) (3,0)

y-intercept: $f(0)=3$

$\mathbf{(0,3)}$

domain: $(-\infty,\infty)$

range: $(\mathbf{-1},\infty)$



b. $g(x)=x^2+3x-4$

Axis of symmetry: $\mathbf{x=-\frac{3}{2}}$

Vertex: $\left(-\frac{\mathbf{3}}{\mathbf{2}},-\frac{\mathbf{25}}{\mathbf{4}}\right)$

x-intercept(s): $0=x^2+3x-4$

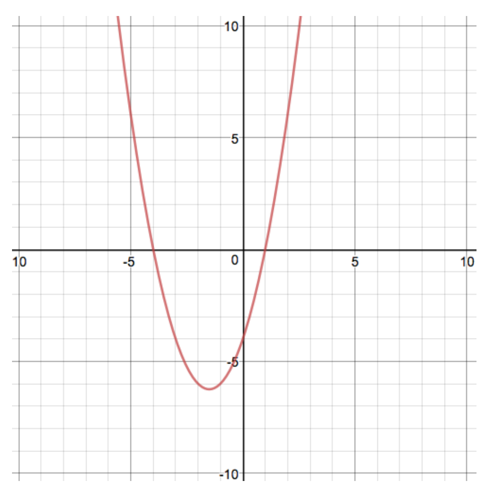
$\mathbf{x=-4,\ \ x=1}$

y-intercept: $g(0)=-4$

$\mathbf{(0,-4)}$

domain: $(-\infty,\infty)$

range: $\left(-\frac{\mathbf{25}}{\mathbf{4}},\infty\right)$



10. *Solve Algebraically*. State your answers in simplest radical form and use standard form for complex solutions.

a. $(x+3)^2+5=1$

$(x+3)^2=-4$

$x+3=\pm\sqrt{-4}$

$x+3=\pm 2i$

$\mathbf{x=-3\pm 2i}$

b. $2x^2+5x=x^2+4$

$x^2+5x-4=0$

$x=\dfrac{-5\pm\sqrt{25-4(1)(-4)}}{2(1)}$

$\mathbf{x=\dfrac{-5\pm\sqrt{41}}{2}}$

c. $3(x^2-x)=x-4$

$3x^2-3x=x-4$

$3x^2-4x+4=0$

$x=\dfrac{4\pm\sqrt{(-4)^2-4(3)(4)}}{2(3)}$

$x=\dfrac{4\pm\sqrt{-32}}{6}=\dfrac{4\pm 4i\sqrt{2}}{6}=\mathbf{\dfrac{2}{3}\pm\dfrac{2\sqrt{2}}{3}i}$

d. $(x+2)(x-1)=6$

$x^2+x-2=6$

$x^2+x-8=0$

$x=\dfrac{-1\pm\sqrt{1^2-4(1)(-8)}}{2(1)}$

$\mathbf{x=\dfrac{-1\pm\sqrt{33}}{2}}$

11. *Solve.* State your answer exactly, then round to 2 decimal places.

a. $2+4(5^{x+3})=10$

$4(5^{x+3})=8$

$(5^{x+3})=2$

$x+3=\log_5 2$

$x=\log_5 2-3$

$\mathbf{x\approx -2.57}$

b. $10e^{4t}=12$

$e^{4t}=\dfrac{12}{10}$

$4t=\ln\left(\dfrac{12}{10}\right)$

$t=\dfrac{1}{4}\left(\ln\left(\dfrac{6}{5}\right)\right)$ *or* $\dfrac{\ln(6/5)}{4}$

$\mathbf{t\approx 0.05}$

c. $\log_3(4x+5)=3$

$4x+5=3^3$

$4x+5=27$

$4x=22$

$x=\dfrac{22}{4}=\dfrac{11}{2}=\mathbf{5.5}$

d. $4+2\log(x-5)=12$

$2\log(x-5)=8$

$\log(x-5)=4$

$x-5=10^4$

$x=10^4-5$

$\mathbf{x=9995}$

12. Find the domain of each function. Write your answers in interval or set builder notation.

a. $f(x) = \sqrt{x}$

$x \geq 0$

$[\mathbf{0}, \infty)$

Or $\{\boldsymbol{x} | \boldsymbol{x} \geq \mathbf{0}, \boldsymbol{x} \in \mathbb{R}\}$

b. $g(x) = \sqrt{3x + 10}$

$3x + 10 \geq 0$

$x \geq -\frac{10}{3}$

$\left[-\frac{\mathbf{10}}{\mathbf{3}}, \infty\right)$

$\boldsymbol{or} \left\{\boldsymbol{x} \middle| \boldsymbol{x} \geq -\frac{\mathbf{10}}{\mathbf{3}}, \boldsymbol{x} \in \mathbb{R}\right\}$

c. $h(x) = 3x^2 + 4$

$(-\infty, \infty)$

Or

$\{\boldsymbol{x} | \boldsymbol{x} \in \mathbb{R}\}$

d. $j(x) = e^x$

$(-\infty, \infty)$

Or

$\{\boldsymbol{x} | \boldsymbol{x} \in \mathbb{R}\}$

e. $k(x) = \log(x)$

$x > 0$

$(\mathbf{0}, \infty)$

Or $\{\boldsymbol{x} | \boldsymbol{x} > \mathbf{0}, \boldsymbol{x} \in \mathbb{R}\}$

f. $m(x) = \log_4(4x + 2)$

$4x + 2 > 0$

$x > -\frac{1}{2}$

$\left(-\frac{\mathbf{1}}{\mathbf{2}}, \infty\right)$

Or $\left\{\boldsymbol{x} \middle| \boldsymbol{x} > -\frac{\mathbf{1}}{\mathbf{2}}, \boldsymbol{x} \in \mathbb{R}\right\}$

13. Answer the following problems algebraically and show your work to verify that your answers are not coming from the graph.

Sweet Harmony Crafts has determined that when x hundred Dobros are built, the average cost per Dobro can be estimated by

$$C(x) = 0.1x^2 - 0.7x + 2.425,$$

where $C(x)$ is in hundreds of dollars.

a. What is the cost for 600 dobros?

See Video for detailed solutions. (The link is on the Math 95 webpage)

$$x = 6$$

$$C(6) = \mathbf{\$182.5}$$

b. When will the cost be $300 (which is 3 hundred dollars)?

$$3 = 0.1x^2 - 0.7x + 2.425$$

$$x = \frac{7 \pm \sqrt{72}}{2}$$

Throw out negative answer

$$x = 7.74\ hundred = \boldsymbol{774\ dobros}$$

c. What is the minimum average cost per dobro and how many dobros should be built in order to achieve that minimum?

The minimum happens at the vertex… so find the vertex!

$$x = 3.5$$

This means that 350 dobros are needed.

$$C(3.5) = 1.2$$

This means that the cost will be $120

14. The population of Austria was about 8.2 million in 2009, and the exponential growth rate was 0.052% per year.

a. Write an exponential function describing the population of Austria t years after 2009.

$$P(t) = 8.2e^{.052t}$$
$$(see\ formula\ sheet\ for\ general\ form)$$

b. What will the population be in 2020? In 2050?

Note that this is years after 2009

$$2020 \rightarrow \quad t = 11$$
$$P(11) = 14.53\ million$$
$$2050 \rightarrow \quad t = 41$$
$$\boldsymbol{P(41) = 69.14\ million}$$

c. When will the population reach 9 million?

$$9 = 8.2e^{.052t}$$
$$\frac{9}{8.2} = e^{.052t}$$
$$\ln\left(\frac{9}{8.2}\right) = .052t$$
$$\frac{\ln(9/8.2)}{.052} = t$$
$$t \approx 1.79\ years$$
$$\boldsymbol{During\ 2010, the\ population\ should\ have\ reached\ 9\ million}$$

d. What is the doubling time?

$$2 = e^{.052t}$$
$$\ln 2 = .052t$$
$$\frac{\ln 2}{.052} = t$$
$$\boldsymbol{t \approx 13.33\ years}$$

Math 95

Final formula Sheet

Quadratic formula

$$x = -\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Vertex formula

$$\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$$

Interest Formulas

$$A = Pe^{rt} \qquad A = P\left(1 + \frac{r}{n}\right)^{nt} \qquad A = P(1 + r)^t$$

Exponential Growth and Decay:

$$P(t) = P_0 e^{kt}$$