# 10-1: Composite and Inverse Functions

Functions are a key component in the applications of algebra. When working with functions, we can perform many useful operations such as addition and multiplication of functions. In this lesson, we will learn about an important operation called the "Composition of two functions".

*Review* Use $f(x) = x^2 + 3x$ to find the following.

a) $f(5) =$

b) $f(-5) =$

c) $f(3a) =$

d) $f(3 + a)$=

## Composite Functions

Let's return to a previous example:

*Example 1:*

*I have decided to open a coffee stand called "Simply Lattes!" I will charge \$2.50 per Latte. My overhead will be \$175 per day. Each cup of coffee will cost \$.75 to make.*

From this situation we have the following functions:

Revenue: $R(n) = 2.5n$

Cost: $C(n) = .75n + 175$

Where $n$ represents the number of lattes sold.

After being open for several months, I have discovered that the number of lattes sold is a function of the high temperature, $t$, and it can be described as follows:

$$n(t) = 240 - 2t$$

*Use this function to find new functions for Revenue, Cost, and Profit* ***in terms of t.***

Revenue: Cost: Profit:

*Composition Function*: a composition function defines one function in terms of another.

The ***composition of the function f with g*** is defined as follows:

$$(f \circ g) = f(g(x))$$

("*f* of *g* of *x*")

The ***domain of the composite function f*** $\circ$ ***g*** is the set of all *x* such that

1. *x* is in the domain of *g* and
2. g(x) is in the domain of *f*

*Example 2*: Use the following pair of functions to find the given compositions.

$f(x) = 2x - 5$

$g(x) = x^2 + 4$

a) $(f \circ g)(2)$

b) $(g \circ f)(2)$

c) $(f \circ g)(x)$

d) $(g \circ f)(x)$

*Example* Use the table to find the following:

$(g \circ f)(-3)$

$(f \circ g)(-3)$

| $x$ | -4 | -3 | -2 | -1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|---|
| $f(x)$ | -6 | -4 | -2 | 0 | 2 | 4 | 6 | 8 |
| $g(x)$ | 15 | 10 | 5 | 0 | -5 | -10 | -15 | -20 |

## Inverse Functions

Some pairs of functions because they "undo" each other.

*Example 3:*

I have decided to sell some merchandise at *Simply Latte's*. I will mark everything up 50% of what I buy it for. This gives me the following function for my sales price:

$$m(x) = 1.5x$$

Where $x$ is the wholesale price at which I will buy the items.

To boost my sales, I've decided to have a sell all my merchandise for $\frac{2}{3}$ of the regular price. This gives me the following function for my sale price:

$$s(x) = \frac{2}{3}x$$

Complete the following table for several items.

| Wholesale: $x$ | Reg. Price: $m(x)$ | Sale Price: $s(x)$ |
|---|---|---|
| $4 | | |
| $12 | | |
| $7 | | |

Since the sale price depends on the regular price, find the following compositions:

$s(m(x))$

$m(s(x))$

The functions $m(x)$ and $s(x)$ are called ________________ functions because

$$s(m(x)) = m(s(x)) = x$$

***Inverse Functions***

Two functions $f$ and $g$ are inverse functions if

$f(g(x)) = x$ for all $x$ in the domain of $g$

*And*

$g(f(x)) = x$ for all $x$ in the domain of $f$

In this case we say the function $g$ is the **inverse of the function *f*.**

*<u>Example4:</u>* Verify that the following pairs of functions are inverse functions.

a) $f(x) = 2x + 3$ and $g(x) = \frac{x-3}{2}$

b) $f(x) = 2x^2$ and $g(x) = \sqrt{\frac{x}{2}}$

c) $f(x) = 4x - 8$ and $g(x) = \frac{x}{4} + 2$

## Finding The Inverse of a Function

The inverse of a function reverses the coordinates of the function. If $f$ is the set of ordered pairs $(x,y)$, then $f^{-1}$ is the set of ordered pairs $(y,x)$.

***Finding the Inverse of a Function:***

1. Replace *f(x)* with y.
2. Interchange $x$ and $y$.
3. Solve for $y$
4. If the inverse is a function, replace $y$ with $f^{-1}(x)$
5. Verify by showing $f\left(f^{-1}(x)\right) = f^{-1}(f(x))$

*Example5: Find the inverse of these functions.*

*a)* $f(x) = x + 7$

*b)* $f(x) = 2x + 6$

*c)* $g(x) = 8x^3 + 1$

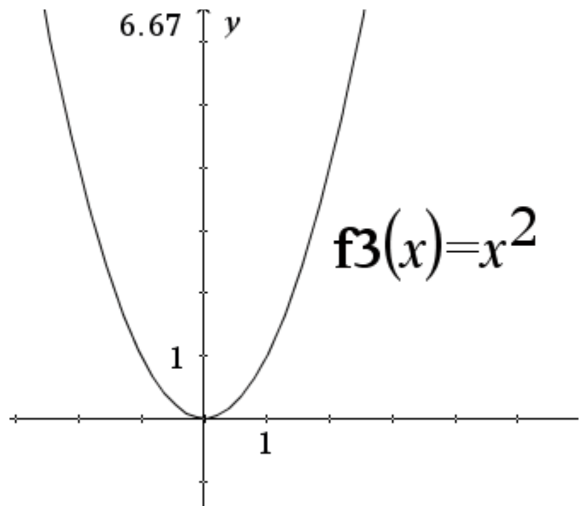
# The Horizontal Line Test and One-to-One Functions.

Not all functions have inverse *functions*. For a function to have an inverse it must be **one-to-one**.

Def. A **one-to-one** function must have different outputs for every input.

*Example 2*: Does $f(x) = x^2$ have an inverse function? Complete the coordinates of *f(x)* and flip them for the inverse.

| $f(x) = x^2$ | $f^{-1}(x)$ (Reverse coordinates) |
|---|---|
| ( -3, ) | |
| (-1, ) | |
| (0, ) | |
| (1, ) | |
| ( 3, ) | |

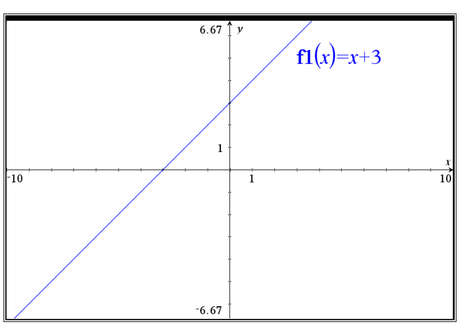



Does the inverse relation $f^{-1}(x)$ above determine a function?

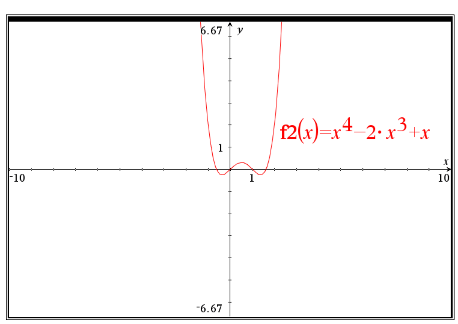
**Horizontal Line Test for Inverse Functions:** A function *f* has an inverse that is a function if there is no horizontal line that intersects the graph of the function *f* at more than one point.

*Example 3:* Determine if the following functions have inverse functions.

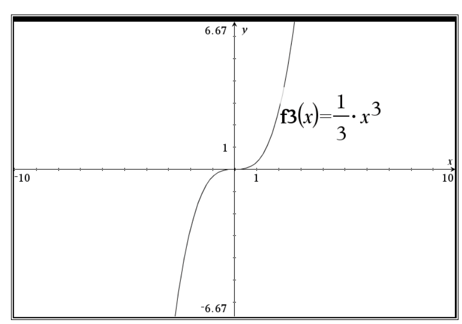
a)




b)




c)



*Example 4:* Graph the functions with their inverses. How does the graph of a function relate to the graph of its inverse?

a) $f(x) = 2x + 3$ and $f^{-1}(x) = \frac{x-3}{2}$

b) $f(x) = 4x - 8$ and $f^{-1}(x) = \frac{x}{4} + 2$

c) $f(x) = x^3 + 5$ and $f^{-1}(x) = \sqrt[3]{x-5}$

*Example 5: Graph the possible inverse of the function below by reversing the x and y coordinates.*
*Is the new graph a function?*