10-2: Exponential Function Introduction

Quadratic functions, like $y = x^2$, are made of a sum of powers of the independent variable. In a quadratic function, the variables are in the base of the power, but what happens if we put the variable in the exponent of a power (*e.g.* $y = 2^x$)? In this case, we have an amazing family of functions called exponential functions.

Definition

A *simple* **exponential function** is of the form

$$f(x) = a(b^x)$$

Where b is positive, and $b \neq 1$. The number a is the initial value, and the number b is the called the *base*.

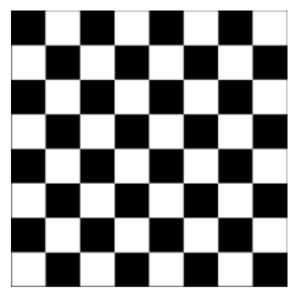
Try These:

Which of the following are exponential functions?

a) $f(x) = 2.5^x$ b) $g(x) = (-3)^x$ c) $h(x) = 2^{-x}$ d) $j(x) = 2(x^3)$

Exploring Exponential Functions and Graphs

Legend has it…



There once was a peasant in India who invented the game of chess. The ruler of the land was so pleased the game, that he wanted to reward the peasant for his creation. So, he asked the man what he would like for his reward. After thinking for a few minutes he said, “I don’t need much. How about if you take my 8×8 chess board and put 1 grain of rice on the first square, 2 grains on the second square, 4 on the third, and continuing doubling the grains of rice on each square

1) Write an exponential function for the grains of rice, G, on a given square n if we begin by numbering the initial square $n = 0$.

2) How many grains of rice were needed to put on the last square?

3) The world production of rice is approximately 500 million metric tons per year. How does the amount of rice *on the last square* compare to the amount of rice produced in a year? (assume $1 grain = \frac{1}{64}\ gram$ and $1\ metric\ ton = 1000kg$)

4) *Extra Challenge:* How many liters of rice would there be on the entire board?

Growth and Decay

Some quantities grow exponentially as the rice in the previous example, while others decrease exponentially.

<u>Example 1</u>: Medicine Dosage

Suppose a patient is given a dosage of 320mg of Penicillin intravenously (so that it reaches the highest concentration immediately). We will assume that half of the drug is absorbed by the body each hour.

a. Find the amount of Penicillin in the patient's blood stream h hours after the drug is administered.

Hours	0	1	2	3	4	5
Amount						

b. Write an equation to describe the amount of Penicillin, P, in the blood stream as a function of time t (in hours).

c. Use your equation to predict the amount of Penicillin in the bloodstream after 10 hours.

<u>Exponential Growth and Decay:</u> The exponential function $y = a(b^x)$ is either a
- *Exponential Growth* equation if $b > 1$ (here we call b the "growth factor"), or
- *Exponential Decay* equation if $0 < b < 1$ (here we call b the "decay factor").

Graphing Exponential Functions

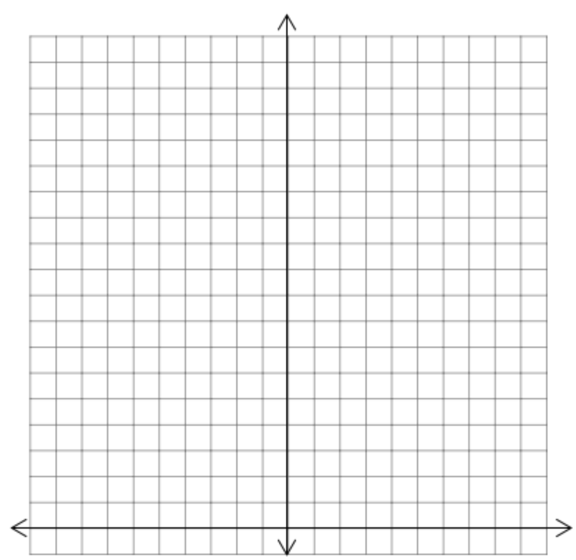
Graph the following functions by hand on the same axes:

$y = 2^x$

$y = 3^x$

$y = \left(\frac{1}{2}\right)^x$

$y = \left(\frac{1}{3}\right)^x$



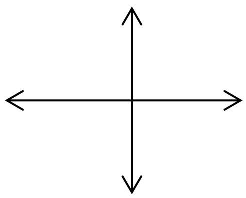
Horizontal Asymptote: *for each of these graphs, the x-axis is called a <u>horizontal asymptote</u> because the graphs get infinitely close to the axis, but they never touch it.*

Translating Exponential Functions

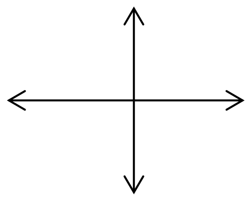
Now let's explore some translations. Predict the shape of the graph, sketch it, then check it on your calculator or desmos (try QR code). Look for patterns in each row.

Changing b

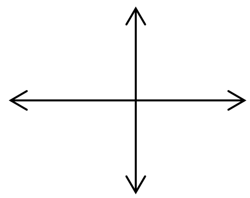
$y = 2^x$



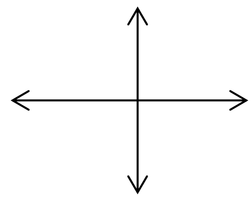
$y = 1.5^x$



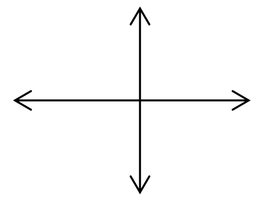
$y = 7^x$



$y = (.3)^x$

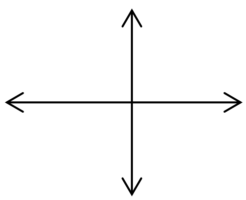


$y = \left(\frac{1}{5}\right)^x$

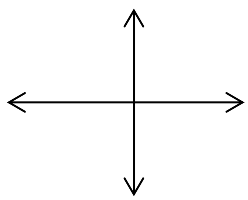


Changing h

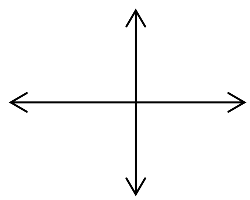
$y = 2^{x+2}$



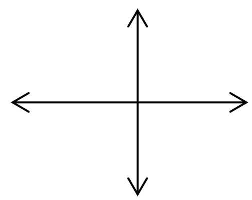
$y = 2^{x+4}$



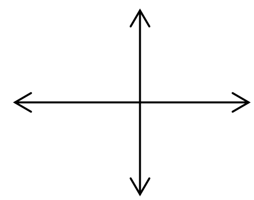
$y = 2^{x-2}$



$y = 2^{x-4}$

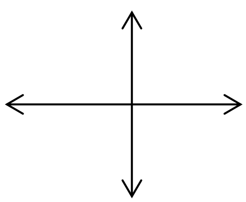


$y = 2^{x-7}$

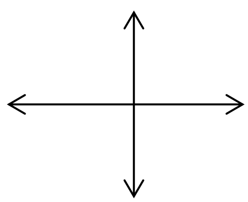


Changing k

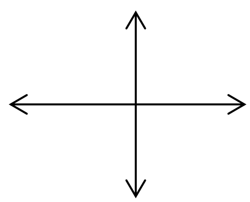
$y = 2^x + 2$



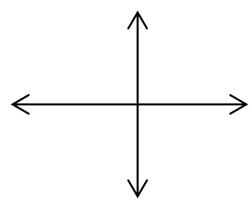
$y = 2^x + 5$



$y = 2^x - 3$



$y = 2^x - 5$



Transformations of Exponential Functions:

Describe how the values of b, h and k affect the graph of $y = a\left(b^{x-h}\right) + k$

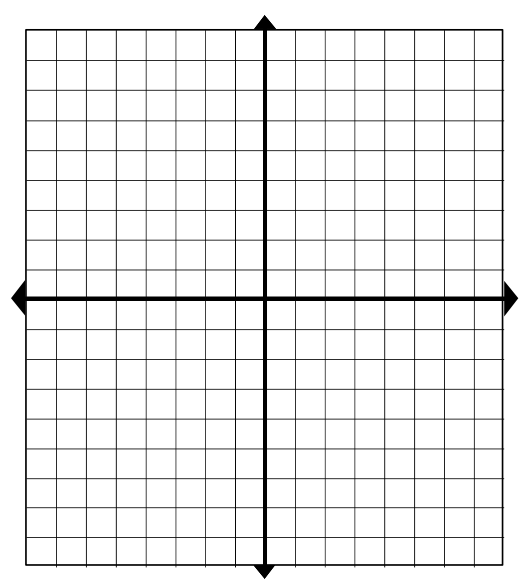
a:

b:

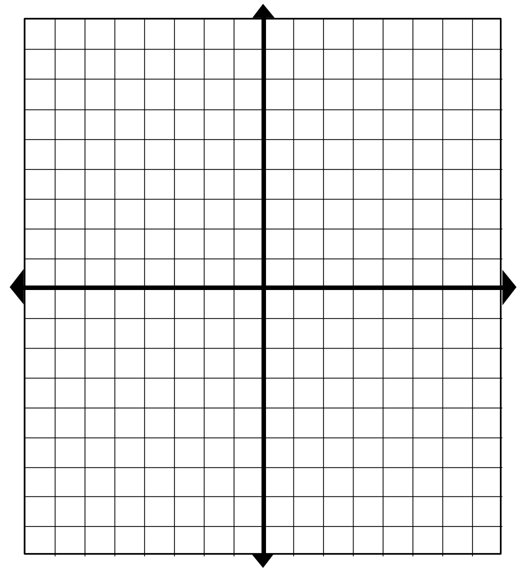
h:

k:

Example 2 Describe how the graph of $y = 2^{x+3} - 5$ differs from the graph of $y = 2^x$, then graph $y = 2^{x+3} - 5$.



Example 3 Graph $y = \left(\frac{1}{3}\right)^{x-2} + 1$



Solving with like bases

When working with equations with exponential expressions (variables in the exponents), we can sometimes use the **like bases property**:

If two equivalent expressions have the same base, then they must have the same exponents.

Example 4: Solve

a) $2^{2x+3} = 2^{15}$

b) $3^{5x} + 4 = 85$

c) $\left(\frac{1}{5}\right)^{3x} = 5^{6}$

d) $\left(\frac{1}{2}\right)^{5x} = 2^{16}$

The Natural Number

One place where exponential functions becomes very useful is computing compound interest. When interest is compounded n times a year, the amount A of an account that had an initial investment of P at an interest rate of r for t years is calculated with the **compound interest formula:**

$$\boldsymbol{A = P\left(1 + \frac{r}{n}\right)^{nt}}$$

Try it:

If you invest \$1,000 in the bank at 6% a year, find the amount after 10 years if it is:

a) Compounded annually:

b) Compounded quarterly:

c) Compounded monthly:

d) Compounded daily:

If we assume that we **invest \$1 at 100% interest for 1 year**, we get the formula

$$A(n) = \left(1 + \frac{1}{n}\right)^n$$

Enter this function into your graphing calculator and observe the table.
What happens to the value of $A(n)$ as n gets very large?

This is called a *limit* of a function because it gets very close to the value, but never reaches it. The limit of the function $A(n) = \left(1 + \frac{1}{n}\right)^n$ is a number "e", where $\boldsymbol{e \approx 2.7182818284}$ **.....**
This is an irrational constant like $\pi \approx 3.14159 \ldots$

The number e is often called the *natural number* or *Euler's number* after the famous mathematician *Euler* (although it was first discovered by John Napier c. 1600).

Example
Graph $f(x) = e^x$ and $g(x) = e^{-x}$