10.5 Exponential and Logarithmic Equations

Changing Logarithmic Bases

We have seen that our calculators can compute logarithms in base 10 and base e, but what if we have a different base? To do this, we can use the *change of base* formula

Change-Of-Base Formula: For any logarithms in base a and b and any positive number M,

$$\log_b M = \frac{\log_a M}{\log_a b}$$

Using the Change-Of-Base Formula:

In general, to compute a log, base b, use one of the following

$$\log_b M = \frac{\log M}{\log b}, \qquad or \qquad \log_b M = \frac{\ln M}{\ln b}.$$

Try It: Use the change of base formula to approximate the value of these logarithms to 3 decimal places. Then check using an exponential.

a. $\log_5 125$ b. $\log_4 2$ c. $\log_5 0.2$ d. $\log_{32} 33$

Properties of Common and Natural Logarithms:

General Properties	Common Logarithms	Natural Logarithms
$\log_b 1 = 0$		
$\log_b b = 1$		
$\log_b b^x = x$		
$b^{\log_b x} = x$		

Exponential Equations

Exponential Equation: an equation with a variable in the *exponent.*

Method 1: *Solving Exponential Equations with Same Bases.*

Example: Solve

a. $2^{4x-2} = 64$ b. $25^{y} = 625$ c. $9^{x+3} = 27$

Method 2: *Solving Exponential equations using logarithms.*

Example: Solve $5^{x+1} + 4 = 30$

Step 1: Isolate the exponential expression

Step 2: Write an equivalent logarithmic equation

Step 3: Solve for the variable

Step 4: Use Change-of-base formula for approximate solution

Example: Solve. Find the exact answer, the approximate it to three decimal places.

a. $3(4^x) = 75$

b. $10e^{2x} - 4 = 26$

c. $4(10^{x+2}) = 28$

I. Logarithmic Equations

Logarithmic Equation: an equation containing a variable in a logarithmic equation.

Example: Solve $3 + \log_4(x+5) = 6$

Step 1: Isolate the logarithm

Step 2: Rewrite in exponential form.

Step 3: Solve

Step 4: Check that solution satisfies
$\log_b M = c$ *where* $M > 0$

Example: Solve.

a. $3 + \log_8(x + 5) = 2$

b. $5\ln(3x) = 45$

b. $\log(3x^2 - 2) = 1$

Application Example: How long will it take \$20,000 to grow into \$100,000 at 9% interest rate compound monthly? Use the compound interest formula where $r = rate, A = Balance, P = Principle, n = \# \; of \; time \; compounded$.

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$