

10.5 Exponential and Logarithmic Equations

Changing Logarithmic Bases

We have seen that our calculators can compute logarithms in base 10 and base e , but what if we have a different base? To do this, we can use the *change of base* formula

Change-Of-Base Formula: For any logarithms in base a and b and any positive number M ,

$$\log_b M = \frac{\log_a M}{\log_a b}$$

Using the Change-Of-Base Formula:

In general, to compute a log, base b , use one of the following

$$\log_b M = \frac{\log M}{\log b}, \quad \text{or} \quad \log_b M = \frac{\ln M}{\ln b}$$

Try It: Use the change of base formula to approximate the value of these logarithms to 3 decimal places. Then check using an exponential.

a. $\log_5 125$

b. $\log_4 2$

c. $\log_5 0.2$

d. $\log_{32} 33$

Properties of Common and Natural Logarithms:

General Properties	Common Logarithms	Natural Logarithms
$\log_b 1 = 0$		
$\log_b b = 1$		
$\log_b b^x = x$		
$b^{\log_b x} = x$		

Exponential Equations

Exponential Equation: an equation with a variable in the *exponent*.

Method 1: *Solving Exponential Equations with Same Bases.*

Example: Solve

a. $2^{4x-2} = 64$

b. $25^y = 625$

c. $9^{x+3} = 27$

Method 2: *Solving Exponential equations using logarithms.*

Example: Solve

$$5^{x+1} + 4 = 30$$

Step 1: Isolate the exponential expression

Step 2: Write an equivalent logarithmic equation

Step 3: Solve for the variable

*Step 4: Use Change-of-base formula
for approximate solution*

Example: Solve. Find the exact answer, the approximate it to three decimal places.

a. $3(4^x) = 75$

b. $10e^{2x} - 4 = 26$

c. $4(10^{x+2}) = 28$

I. Logarithmic Equations

Logarithmic Equation: an equation containing a variable in a logarithmic equation.

Example: Solve

$$3 + \log_4(x + 5) = 6$$

Step 1: Isolate the logarithm

Step 2: Rewrite in exponential form.

Step 3: Solve

Step 4: Check that solution satisfies

$$\log_b M = c \text{ where } M > 0$$

Example: Solve.

a. $3 + \log_8(x + 5) = 2$

b. $5 \ln(3x) = 45$

c. $\log(3x^2 - 2) = 1$

Applications:

Exponential Growth Function is of the form $P(t) = P_0 e^{kt}$

$P_0 =$ initial amount, $k =$ growth rate, $t =$ time

Example: The population of Oregon in 2010 was 3.83 million. In 2018 the population was 4.19 million. Find the exponential growth rate percentage over the 8 years from 2010 to 2018.

Example: How long will it take \$20,000 to grow into \$100,000 at 9% interest rate compound monthly? Use the compound interest formula where $r =$ rate, $A =$ Balance, $P =$ Principle, $n =$ # of time compounded.

$$A = P \left(1 + \frac{r}{n} \right)^{nt}$$