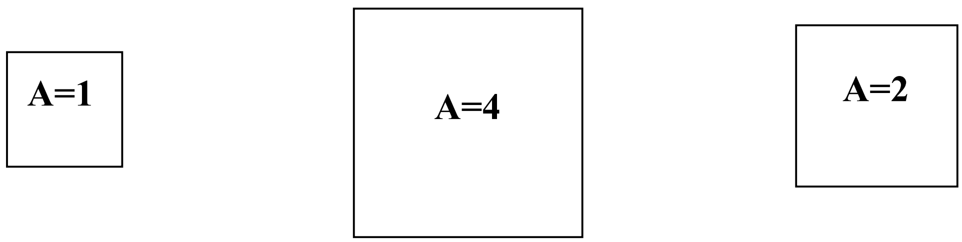
8.1: Simplifying Expressions with Roots

Square roots have played a key role in our understanding of mathematical systems since they were the door to the first *irrational* numbers discovered by the Greeks in the days of Pythagoras. In this lesson we will introduce the concept of square root, cube root, and n^{th} root expressions and functions.

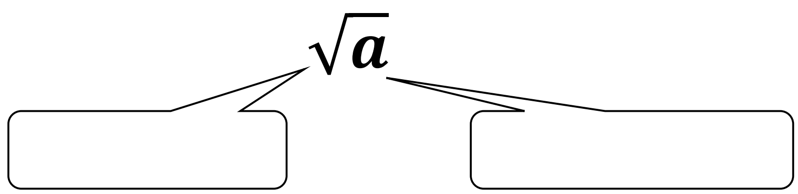
I. Square Roots

Example 1:

Find the length of the side of each square for the given area:



Radical Expression:



Every non-zero number has ________________ square roots.

Example 2: Find the square roots of 36

Definition: The **principal square root** of a is the ______________________, b such that $b^2 = a$.

Example 3: Evaluate:

a) $\sqrt{9}$ b) $-\sqrt{0.49}$ c) $\sqrt{\frac{36}{49}}$

d) $\sqrt{100-36}$ e) $\sqrt{100}-\sqrt{36}$ f) $\sqrt{-16}$

g) $\sqrt{(5)^2}$ h) $\sqrt{(-5)^2}$ g) $\sqrt{x^2}$

II. Simplifying Square Roots

<u>Example 6</u>: Simplify $\sqrt{x^2}$, then verify by graphing $y = \sqrt{x^2}$

<u>Example 7:</u> Simplify

a) $\sqrt{(-3)^2}$

b) $\sqrt{(x-3)^2}$

c) $\sqrt{36x^8}$

Roots are the inverse of Powers! So, to help us understand roots, let's find some powers. Fill in the table below

n	1	2	3	4	5	6	7	8	9	10	11	12	13
n^2													
n^3													
n^4													
n^5													

III. Cube Roots

Find the value of x that makes the following equations true:

a) $x^3 = 8$

b) $x^3 = -27$

> ***<u>Definition:</u>*** A **cube root** of a real number a is written as $\sqrt[3]{a}$.
> If $b^3 = a$, then $\sqrt[3]{a} = b$

<u>Example 10</u>: Find the roots

a) $\sqrt[3]{1}$ b) $\sqrt[3]{8}$ c) $\sqrt[3]{-8}$ c) $\sqrt[3]{-64}$

<u>Example 9:</u> Simplify

a) $\sqrt[3]{a^3}$

b) $\sqrt[3]{-125x^3}$

IV. *N*th Roots

We can extend this idea to any positive power.

<u>Definition:</u> The *n*th root of a is written as follows:

$$\sqrt[n]{a}$$

In general, if $b^n = a$, then $\sqrt[n]{a} = b$. The number n is called the ***index.***

- If n is odd, then the root is an **odd root** which has the same sign as a
- If n is even, then it is a **principle root** and it must be positive.

<u>Example 10:</u> Find the indicated root.

a) $\sqrt[4]{16} =$

b) $\sqrt[4]{-16} =$

c) $-\sqrt[4]{16} =$

d) $\sqrt[5]{-81} =$

Simplifying *n*th-roots

For any real number *a*:

1. If n is even, $\sqrt[n]{a^n} =$

2. If n is odd, $\sqrt[n]{a^n} =$

Reviewing Power Rules

Since radicals are the opposite of powers, we need to remember some power rules.

$$(a^m)^n = a^{mn}, \qquad (ab)^m = a^m b^n$$

When simplifying a radical,

1. First rewrite the radicand as a power equal to the index, then
2. Simplify the expression by canceling the power with the root, then
3. Positive indices simplifying to Odd powers need Absolute Value.

Example 11: Simplify

a) $\sqrt[4]{16a^4}$

b) $\sqrt[5]{-32x^5}$

c) $\sqrt{49x^4y^6}$

d) $\sqrt[3]{27a^3b^{12}}$