8.2: Simplifying Radical Expressions

I. The Product Rule for Radicals

Multiply $\sqrt{25} \cdot \sqrt{4} =$

Rule: Radical Product Rules

If $\sqrt[n]{a}$ and $\sqrt[n]{b}$ are real numbers, then

$$\sqrt[n]{a} \cdot \sqrt[n]{b} =$$

Example 1:

a. $\sqrt{2} \cdot \sqrt{5} =$

b. $\sqrt{x+5} \cdot \sqrt{x-5} =$

c. $\sqrt[3]{2} \cdot \sqrt[3]{-4} =$

d. $\sqrt[5]{3x} \cdot \sqrt[5]{2x^4} =$

II. Using Factoring and the Product Rule to Simplifying Radicals

An nth-index radical is **simplified** if the radicand has no factors that are perfect *n*th powers.

Example 2: Simplify $\sqrt{18}$

Method 1 Steps:

1. Factor the radicand to include one perfect *n*th power.
2. Use the product rule to take the *n*th power of each factor.
3. Find the *n*th root of the perfect *n*th power.

Method 2 Steps:

1. Write prime factorization of radicand.
2. Break into 2 radicals (one that can be simplified)
3. Simplify the one radical and simplify the "left-overs" under the radical.

Example 3: *Use the Radical Product rule to simplify the radicals.*

a) $\sqrt{50}$

b) $\sqrt{80}$

c) $\sqrt[3]{32}$

d) $\sqrt[5]{32xy^5}$

Example4: Simplify using the radical product rule. Use Absolute value signs as needed.

a) $\sqrt{x^5y^6z^{13}}$

b) $\sqrt[3]{54x^3y^8z^{22}}$

III. Dividing Radical Expressions

Try this: Use rational exponents to compute the following expression

$$\sqrt[3]{\frac{27}{8}}$$

Rule: Radical Quotient Rule

If $\sqrt[n]{a}$ *and* $\sqrt[n]{b}$ *are real numbers and* $b \neq 0$, *then*

$$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$$

Example 1: Simplify each expression by simplifying the fraction, then use the radical quotient rule.

a. $\sqrt[3]{\frac{16}{27}}$

b. $\sqrt[3]{\frac{24}{125}}$

c. $\sqrt{\frac{9x^3}{y^{10}}}$

d. $\sqrt[3]{\frac{8y^7}{x^3}}$

Example 5: Divide and Simplify. When you have radicals separate, combine them first. Then try to simplify.

a. $\frac{\sqrt{40x^5}}{\sqrt{2x}}$

b. $\frac{\sqrt[3]{48x^8y}}{\sqrt[3]{6xy^4}}$