8.3: Simplifying Rational Exponents

Exponents are a powerful tool that allow us to simplify repeated multiplication. When working with exponents, we have several special properties such as

$$b^m \cdot b^n = b^{m+n}, \quad \frac{b^m}{b^n} = b^{m-n}, \quad (ab)^m = a^m b^m, \quad \left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}, \quad b^{-m} = \frac{1}{b^m}, and \quad b^0 = 1$$

for integers $a, b > 0$, and $a \neq 1, b \neq 1$, and integer exponents m and n. In this lesson, we will consider exponents that are not integers. That is, what does it mean to evaluate $8^{4/3}$?

I. Defining Rational Exponents

Let's begin by reviewing some applications of the exponent properties above.

Review: Simplify

a) $(a^2b^6)^2$

b) $-(3x^2)^4x^{-3}$

c) $\frac{x^2y^4}{x^5y^2z^{-4}}$

Consider this: Apply the properties of exponents to the following expressions

a. $9^{\frac{1}{2}} \cdot 9^{\frac{1}{2}} =$ or… $\left(9^{\frac{1}{2}}\right)^2 =$ So, $9^{\frac{1}{2}} =$

b. $8^{\frac{1}{3}} \cdot 8^{\frac{1}{3}} \cdot 8^{\frac{1}{3}} =$ So, $\left(8^{\frac{1}{3}}\right)^3 =$ So $8^{\frac{1}{3}} =$

c. Solve for x: $125^{\frac{1}{3}} = x$

***Definition:* If a is a real number and $n \geq 2$ is an integer, then**

$$a^{\frac{1}{n}} =$$

Example 1: Simplify if possible

a. $27^{\frac{1}{3}}$

b. $(-32)^{\frac{1}{5}}$

c. $(36x^3y^6)^{\frac{1}{2}}$

Definition: If a and $\frac{m}{n}$ are real numbers, and $\frac{m}{n} > 0$, then

$$a^{\frac{m}{n}} = \sqrt[n]{a^m}$$

or

$$a^{\frac{m}{n}} = \left(\sqrt[n]{a}\right)^m$$

Example 2: Write in radical notation and simplify if possible.

a. $8^{\frac{4}{3}}$

b. $25^{\frac{3}{2}}$

c. $(27x^3)^{\frac{2}{3}}$

Example 3: Write with rational exponents

a. $\sqrt[5]{7^7}$

b. $\left(\sqrt[5]{7}\right)^7$

c. $\left(\sqrt[3]{13xy}\right)^{25}$

Definition: If a and $\frac{m}{n}$ are real numbers, and $\frac{m}{n} \neq 0$, then

$$a^{-\frac{m}{n}} = \frac{1}{\sqrt[n]{a^m}}$$

Example 4: Rewrite each expression in radical form.

a. $49^{-\frac{1}{2}}$

b. $(31xy)^{-\frac{5}{21}}$

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II. Using Properties of Rational Exponents

****** Key Point: the properties of exponents apply to <u>all real exponents</u> whether they are fractions or integers*.**

$$b^m \cdot b^n = b^{m+n}, \quad \frac{b^m}{b^n} = b^{m-n}, \quad (ab)^m = a^m b^m, \quad \left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}, \quad b^{-m} = \frac{1}{b^m}, and \quad b^0 = 1$$

<u>*Example 5:*</u> Use the properties of exponents to simplify and then write in exponential form. Begin by writing them with rational exponents, then simplify.

a. $\sqrt[12]{x^6}$

b. $\sqrt[3]{125x^{30}}$

c. $\sqrt[6]{x^3y^2}$

d. $\sqrt{x} \cdot \sqrt[5]{x^2}$

e. $\sqrt[3]{\sqrt{x^9}}$

<u>*Example 6:*</u> Use the properties of exponents to simplify.

a. $\left(125x^{\frac{6}{5}}y^{\frac{3}{2}}\right)^{\frac{2}{3}}$

b. $\left(\frac{9x^2y^{-\frac{1}{2}}}{x^{-\frac{1}{2}}}\right)^{\frac{2}{5}}$

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