8.4: Add, Subtract, and Multiply Radicals

I. Adding and Subtracting Radical Expressions

Example 1: Simplify

a. $5x + 3x =$

b. $5\sqrt{2} + 3\sqrt{2} =$

c. $4 + 5\sqrt[3]{17} - 3\sqrt[3]{17}$

Rule: When simplifying radical expressions, we can only add or subtract __________radicals.

*Example 2:*Simplify

a. $4\sqrt{3} - 7\sqrt{3}$

b. $5\sqrt[3]{5} - 3\sqrt{5} + 7\sqrt[3]{5}$

c. $3\sqrt{2} + 7\sqrt{3} + \sqrt{2} - 12\sqrt{3}$

d. $3y\sqrt{2} + 3x\sqrt{3} + y\sqrt{2} - x\sqrt{3}$

***If the radicals are not alike, try simplifying all radicals first.*

Example 3: Simplify

a. $\sqrt{2} + \sqrt{8}$

b. $\sqrt{12} + \sqrt{75}$

c. $3\sqrt{32} - 5\sqrt{18}$

d. $6\sqrt{12x} - 7\sqrt{27x}$

e. $3\sqrt[3]{24} - 5\sqrt[3]{81}$

f. $5\sqrt[3]{x^2y} + \sqrt[3]{27x^5y^4}$

II. Multiplying and simplifying radicals

We have been using the radical product rule to pull radicals apart and simplify, but it also tells us how we can multiply radicals.

Try it out Multiply and simplify.

a. $5\sqrt{2} \cdot 3\sqrt{6}$

b. $6\sqrt[3]{18x^2} \cdot 4\sqrt[3]{6x^5}$

What if we get a large radical that is difficult to find the largest perfect power factor? The key is to use prime factorization first. When you keep the radicand written as in factored form, you can treat the numbers (coefficients) the same way that you treat the variables.

c. $\sqrt{12x^3y^2} \cdot \sqrt{26x^2y} \cdot \sqrt{39xy^2}$

III. Products and Quotients of Two or More Radical Terms

When multiplying or dividing radical expressions with multiple terms, we use the same techniques as we do when working with polynomial expressions.

Example 4: Multiply and Simplify

a. $\sqrt{2}\left(x - \sqrt{4}\right)$

b. $\sqrt[3]{x^2}\left(\sqrt[3]{x^4} + \sqrt[3]{x}\right)$

c. $\left(5-\sqrt{3}\right)^2$

d. $\left(2\sqrt{3}+\sqrt{5}\right)\left(\sqrt{3}-3\sqrt{5}\right)$

e. $\left(\sqrt{a}+\sqrt{b}\right)\left(\sqrt{a}-\sqrt{b}\right)$

IV. Using Special Products

Example 4e has a special outcome that might look familiar. We will often use special products such as a square of a biniomial and the difference of two squares to help us quickly simplify expressions.

Binomial Squares:

$(a+b)^2 = a^2 + 2ab + b^2$ $\qquad$ $(a-b)^2 = a^2 - 2ab + b^2$

Difference of two squares

$$(a+b)(a-b) = a^2 - b^2$$

Note: this last relationship is very useful!

We call the the binomials $(a+b)$ and $(a-b)$ ***conjugates*** *of each other*

Example 5: Use the special products to simplify.

a. $\left(\sqrt{5}+\sqrt{3}\right)^2$

b. $\left(2\sqrt{3}-3\sqrt{5}\right)^2$

c. $\left(\sqrt{117}+\sqrt{116}\right)\left(\sqrt{117}-\sqrt{116}\right)$

