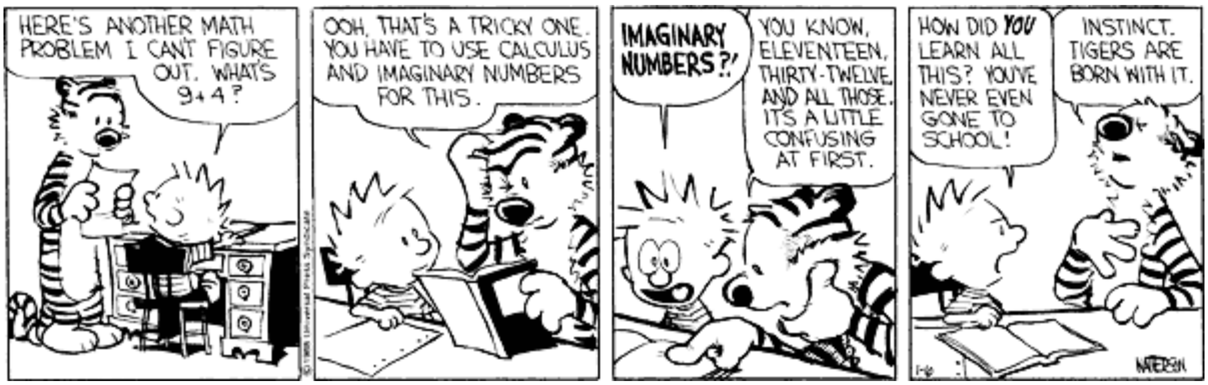
8.8 – Complex Numbers

When working with radicals, we run into problems when we try to take an even root of a negative number. So, we need to develop a way to work with this situation. To do this, we will use the *imaginary number!*



I. The Imaginary Unit *i*

<u>*Try these:*</u> Solve for x

a) $x^2 = 9$ b) $x^2 = 6.25$ c) $x^2 = 2$ d) $x^2 = -1$

All of these have answers that are real numbers except for (d). So, we must define a new type of number that is not real:

<u>***The Imaginary Unit i***</u>

The **imaginary unit** is defined as:

$$i = \sqrt{-1} \qquad \text{where} \qquad i^2 = -1$$

<u>*Example 1:*</u> Simplify

a) $\sqrt{-36}$ b) $\sqrt{-5}$ c) $\sqrt{-20}$

Complex numbers and Imaginary numbers:

- The set of complex numbers is the union of the **real** numbers and the **imaginary** numbers.
- Complex numbers can be written in the form

$$a + bi$$

- With real numbers a and b. We call a the real part and bi the imaginary part.

  If $b = 0$, then $a + bi$ is a ***real*** number.

  If $b \neq 0$, then $a + bi$ is an ***imaginary*** number.

II. Adding and Subtracting Complex Numbers

***** When adding and subtracting Complex numbers, we must combine like terms.**

Example 2 Simplify

a) $(3 + 2i) + (4 + i)$ b) $(-3 + 4i) - (-2 - 9i)$

III. Multiplying Complex numbers

***** When multiplying and dividing Complex numbers, remember that $i^2 =$ __________**

Example 3 Simplify

a) $(3 + 2i)(4 + i)$ b) $(-3 + 2i)(4 - 6i)$

c) $(3 + 2i)(3 - 2i)$ d) $\sqrt{-5} \cdot \sqrt{-7}$

IV. Conjugates and Division

Complex Conjugates *The conjugate of* $\boldsymbol{a+bi}$ *is* ______________________

Multiplying conjugates: Simplify

a. $(5-2i)(5+2i)$

b. $(a+bi)(a-bi)$

- ***Complex Conjugate Product Rule:*** $(a+bi)(a-bi) =$

Dividing Complex Numbers: When dividing complex numbers, make the denominator real and distribute the denominator

Example 4: Divide and simplify to the form $a+bi$

a) $\frac{6+2i}{4-3i}$

b) $\frac{3i-4}{5i}$

V. Powers of *i*

Example 5: Simplify

$i^1 =$

$i^2 =$

$i^3 =$

$i^4 =$

$i^5 =$

$i^6 =$

$i^7 =$

$i^8 =$

Steps for simplifying powers of i:

1. Express the given power of *I* in terms of i^{-2}.
2. Replace i^{-2} with -1 and simplify.

Do you see a pattern? Try these:

$i^{14} =$ $i^{25} =$ $i^{51} =$