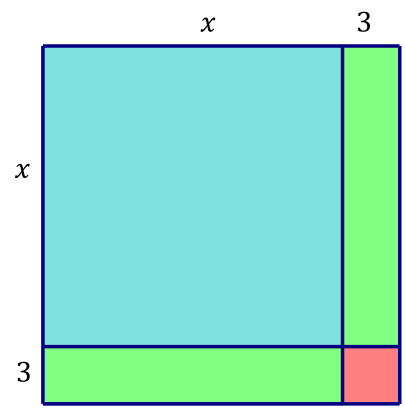
9-2: Completing the Square Method

We have seen four methods for solving quadratic equations so far: factoring, graphing, and the square root methods. These all have some plusses and minuses. Now we will learn a method that will give us the exact answer for any quadratic equation.

Solving quadratic equations by completing the square

What if a quadratic equation cannot be factored and it has an x term (so we can't use the principle of square roots) ?

This is a good question. For this we need to know how to "Complete the Square"



Consider the geometric model shown to the right. We want to describe the area of this square in multiple ways.

Try this: Find an algebraic expression to describe the total area of this square.

Can you find a different expression for the total area of the large square?

a) Using the length of the side, we get the factored form:

$$(x+3)^2$$

b) Using the areas of the four rectangles we get the expanded form:

$$x^2+3x+3x+9\,.$$

c) From which we get the final simplified version

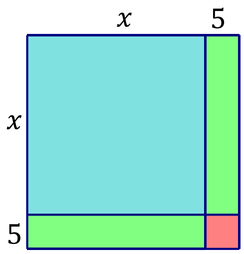
$$x^2+3x+3x+9 \;=x^2+6x+9.$$

This gives us a nice visual proof that $(x+3)^2=x^2+6x+9$.

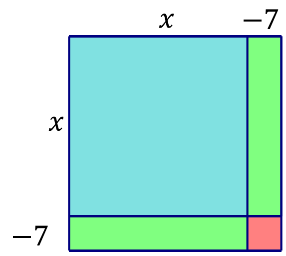
Try these:

For each square, write the area represented in factored, expanded, and simplified form.

a.



b.

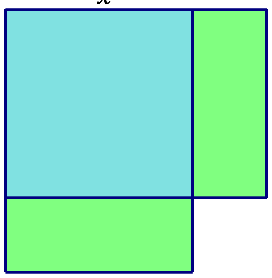


Missing squares...

These "squares", are missing something. As they are, we define the area as the square of a binomial (i.e. factored form.) For each shape,

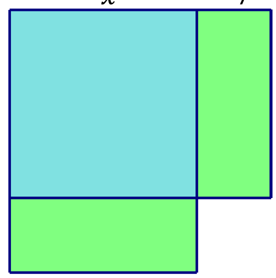
a) write an expanded expression for the area.

b) Then decide the area of the corner square what needs to be added to make a complete square.



$Current\ Area =$

$"Missing"\ area =$



$Current\ Area =$

$"Missing"\ area =$

To generalize this: **To complete the square for $x^2 + bx$, add $\left(\frac{b}{2}\right)^2$.**

So, How does this help? When we complete the square, we can factor one side and use the principle of square roots.

Example. Solve by completing the square.

a) $x^2 + 6x = 16$

b) *Note: First move constants to one side.*

$x^2 - 10x - 3 = 4$

c) *Note: The leading coefficient must be 1*

$2x^2 + 24x = 8$

d) *Note: Don't be afraid of fractions*

$2x^2 + x = 3$

e) *Really…don't be afraid of fractions!* $3x^2 + 2x - 5 = 0$

Summary of Solving Quadratic Equations by Completing the Square

**Note: The goal of completing the square is to make a perfect-square trinomial that can be factored so you can use the square root method.*

Steps:	*Example 3:*
	$2x^2 + 12x - 5 = 9$
1. Isolate the variable terms	$2x^2 + 12x = 14$
2. Divide by leading coefficient	$\frac{2x^2 + 12x}{2} = \frac{14}{2}$ $x^2 + 6x = 7$
3. Complete the Square by adding $\left(\frac{b}{2}\right)^2$ *to both sides of the equation*	$\left(\frac{b}{2}\right)^2 = \left(\frac{6}{2}\right)^2 = 9$ $x^2 + 6x + 9 = 7 + 9$ $x^2 + 6x + 9 = 16$
4. Factor the polynomial	$(x+3)^2 = 16$
5. Use *Square Root Method* and write two equations	$\sqrt{(x+3)^2} = \pm\sqrt{16}$
	$x + 3 = 4$ *or* $x + 3 = -4$
6. Solve both equations	
7. Check answers	$x = 1$ *or* $x = -7$

You Try it: Solve by completing the square.

a) $x^2 + 6x - 3 = 7$

Steps for Completing the Square

1. Isolate the Variable Terms
2. Set Leading Coefficient to 1
3. Complete the square: $\left(\frac{b}{2}\right)^2$
4. Factor and solve.

b) $x^2 + 3x = 2$

c) $2x^2 + 8x - 3 = 7$

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