9.3: The Quadratic Formula

When we need to solve a quadratic equation, then

- we can first try *graphing* but this method is not always exact; or
- we can use the *square root method* if there is no x-term or if we have a perfect square trinomial;
- we can *factor* and use the zero product property but not all quadratics can be factored, or
- we can use *completing the square* which will solve any quadratic equation.

However completing the square can be a little cumbersome, so we will need to find a way to quickly solve any quadratic equation.

Finding and using the "Quadratic Formula"

Review: Solve by completing the square

$$2x^2 + x - 6 = 0$$

The big question: Can we find a general formula by solving a generic quadratic equation?

In other words, we want to solve $ax^2 + bx + c = 0$

Quadratic Formula: For any quadratic equation in the form $ax^2 + bx + c = 0$, the solution(s) can be found using this formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Example 1: *Solve using the quadratic formula.* $2x^2 + x - 6 = 0$

a = 2 *b = 1* *c= -6*

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(1) \pm \sqrt{(1)^2 - 4(2)(-6)}}{2(2)}$$

$$x = \frac{-1 \pm \sqrt{1+48}}{4}$$

$$x = \frac{-1 \pm \sqrt{49}}{4}$$

$$x = \frac{-1+7}{4} \quad \text{or} \quad x = \frac{-1-7}{4}$$

$$x = \frac{6}{4} \quad \text{or} \quad x = \frac{-8}{4}$$

$$x = \frac{3}{2} \quad \text{or} \quad x = -2$$

Example 2: Solve using the quadratic formula.

$$2x^2 + 3x - 5 = 0$$

Consider this: Could the equation in Example 2 have been solved by factoring?

Try these. *Solve using the quadratic formula. Find the exact solutions first, then use your calculator to approximate the solutions to three decimal places.*

*** Remember: the equation must be in standard form before using the quad. formula*

1. $2x^2 + 3x = 4$

2. $9x^2 - 12x + 4 = 0$

3. $2x^2 + 4x = -2$

4. $2t^2 + 4 = 5t$

Summary of Algebraic Methods for Solving Quad. Equations:

To solve $ax^2 + bx + c = 0$

1. If *b=0* …………………… Use the ***square root method***
2. If it's *factorable*………….. ***Factor*** and use Zero-product property
3. If it's *not factorable*……….Use ***Quadratic Formula*** or ***complete the square***

Example 3 Decide what method would be best to solve the following quadratic equations, then write the *first step* to solve the equation (you don't need to solve it.)

a) $5x^2 + 25x = 0$

b) $12x^2 - 3x + 7 = 0$

c) $35x^2 - 20 = 0$

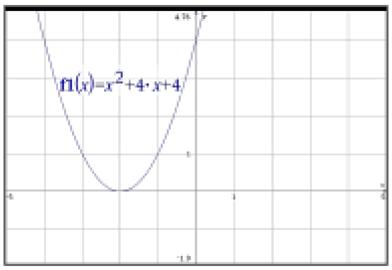
The Discriminant

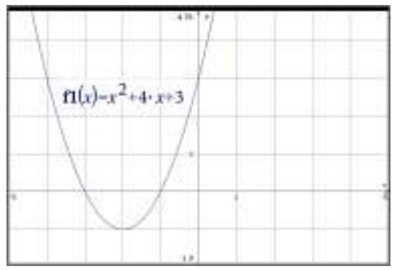
When working with quadratic equations, it is useful to know how many real solutions there are.

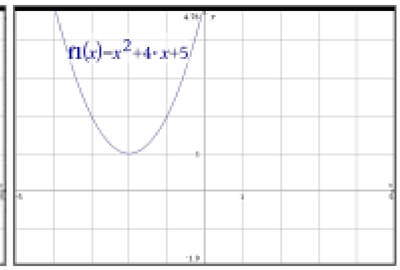
Explore Use the Quadratic Formula to find the *real* solutions of each of these equations.

a) $0 = x^2 + 4x + 4$ b) $0 = x^2 + 4x + 3$ c) $0 = x^2 + 4x + 5$

Graphically: Locate your solutions above on the graphs below.







Consider this: At what point in your process did you realize that you were going to have one, two, or three *real* solutions in the equations above?

The **discriminant** (the part of the quadratic formula under the radical) determines how many solutions a quadratic equation has.

Discriminant = $b^2 - 4ac$

Value of Discriminant ($b^2 - 4ac$ **)**	**Number of Solutions for the equation**
Positive, perfect square number	
Positive, non-square number	
0	
Negative	

Example 2: For each equation,

i) determine what type of number the solutions are,

ii) determine how many real solutions exist, and
iii) confirm your answers by graphing.

a) $4x^2 - 3x - 1 = 0$

b) $5x^2 + 2x + 3 = 0$

c) $9x^2 - 24x + 16 = 0$

d) $2x^2 + 5x + 1 = 0$

Writing Equations from Solutions

We have seen that the zero product rule is quite useful to help us find solutions of equations that can be factored such as $(x-4)(x+5) = 0$. However, we can also use this principle to *build* equations that have more complicated solutions.

Example 3: Find an equation for which the given numbers are solutions.

a) -4 and $\frac{3}{5}$

b) $3\sqrt{2}$ and $-3\sqrt{2}$

c) $3i$ and $-3i$

c) $-1, 0,$ and 2

Example 4: The *Swedish Sisters* coffee shop performed a study and found out if they sell their Vanilla Lattes for \$1.00 they will make no profit because they will not cover their costs and overhead. If they sell their Vanilla Lattes for \$6.50, they will make no profit because no one will buy them! Use these two numbers to make a quadratic function to model their Profit, P, as a function of the Sales price, s.