9.4: Equations Reducible to Quadratic

In this lesson, we will see how to take an equation that is not quadratic and apply quadratic methods to solve it.

Quadratic Form

An equation is in **quadratic form** if it can be written as

$$a(u)^2 + b(u) + c = 0.$$

If an expression is a trinomial, we can recognize that it is in quadratic form if the power on one variable (or expression)term is *twice as large* as the next term's power.

> **Step 1**: Set the equation equal to 0
> **Step 2:** Determine if it is in quadratic form and define the value of u
> **Step 3:** Substitute u into the equation and use quadratic methods to solve for u
> **Step 4:** Re-substitute the original expression for u and solve for original variable.
> **Step 5:** Check answer in original equation.

Try it Determine if the equations are in quadratic form, then define the value of u.

a) $x^4 - 5x^2 + 6 = 0$

b) $(x^2 - 3)^2 + (x^2 - 3) - 5 = 0$

c) $x + 8\sqrt{x} - 9 = 0$

d) $2\,m^{-2} - 5\,m^{-1} - 12 = 0$

e) $t^{\frac{2}{3}} - t^{\frac{1}{3}} - 2 = 0$

f) $\left(10 - \sqrt{x}\right)^2 - 2\left(10 - \sqrt{x}\right) - 35 = 0$

Substitute and Solve

Now let's go the next step and solve these equations using u-substitution.

a) $x^4 - 5x^2 + 6 = 0$

b) $(x^2 - 3)^2 + (x^2 - 3) - 5 = 0$

c) $x + 8\sqrt{x} - 9 = 0$

d) $2\,m^{-2} - 5\,m^{-1} - 12 = 0$

Example **Find the x −intercepts of these functions**

a) $g(t) = t^{\frac{2}{3}} - t^{\frac{1}{3}} - 2$

b) $h(x) = \left(10 - \sqrt{x}\right)^2 - 2\left(10 - \sqrt{x}\right) - 35$