9.5: Applications of Quadratic Equations

In this lesson, we will explore some real-world problems that involve quadratic equations.

When solving problems, break it up into steps like these:

1. Plan and Translate:
    - What information do we know? What do we need to find?
    - What mathematical properties, formulas, or techniques may be helpful?
    - Can you draw and label a picture or diagram for the situation?
2. Process
    - Define variables
    - Substitute known information into equations or formulas and solve equations
    - Use graphs to solve problem or guide your process.
3. Check
    - Substitute solution into formulas or equations to verify.
    - Use graphs to check solutions.

Number Problems

Example The product of two consecutive odd integers is 195. Find the integers.

Area Problems

Start with a Triangle

An architect is designing the entryway of a restaurant. She wants to put a triangular window above the doorway. Due to energy restrictions, the window can only have an area of 120 square feet and the architect wants the base to be 4 feet more than twice the height. Find the base and height of the window.

Try a Rectangle Problem!

Old MacDonald had a farm…. On that farm he had some pigs…. and wants to make a rectangular pen for his pigs against his barn, so he needs to fence in *3 sides of the pen*. He has 80 feet of fencing and he wants to make a pen with the largest amount of area for his pigs.

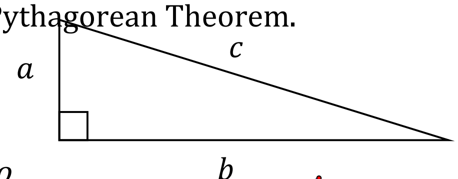
What is the largest area that he can enclose?

What dimensions give him this area?

Pythagorean Theorem

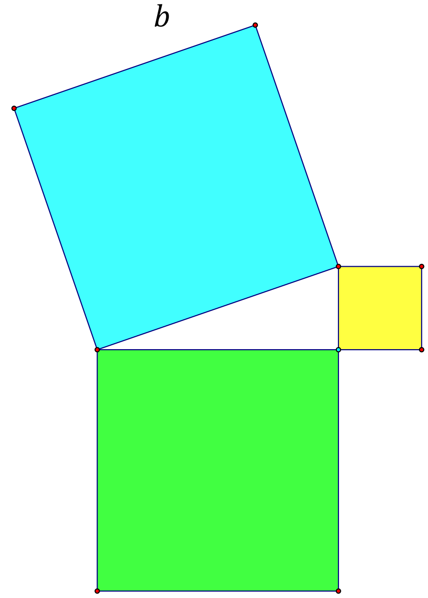
In this lesson we will investigate several applications of radical expressions and equations beginning with one of the most important equations in math, the Pythagorean Theorem.

I. Right Triangles and The Pythagorean Theorem



Pythagorean Theorem: *"In all right triangles, The sum of the area of the squares on the legs is equal to the area of the square on the hypotenuse."*

Algebraically: *If a triangle is a right triangle with legs of length a and b, and a hypotenuse of length c, then…*



Example 1 Find the missing side for the following right triangles. State your answer in simplest radical form.

a) $a = 3,\ b = 4$

b) $a = 10, c = 26$

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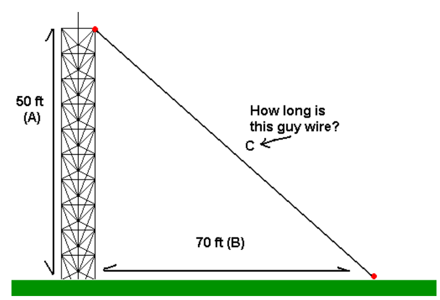
Example 2

a) Suppose a 50 foot **cell tower** is being built, and a guy wire is going to be attached to the ground 70 feet from the base. How long will this wire need to be?



b) Suppose we have a 100 foot **wire** that we want to use, how far from the base should the anchor be put in the ground to attach this 100 ft. guy wire?

Falling Objects

The distance that a free-falling object travels is $s(t) = 16t^2$. If you drop a penny from the top of the roof of the Empire state building at 1250 feet high, how long will it take to reach the ground?

Projectiles

The height of a projectile (an object that is propelled into the air without any means of self-propulsion) is determined by the function

$$s(t) = s_0 + v_0 t - \frac{1}{2} g t^2$$

Where

$$s_0 = intial\ height,\ \ v_0 = initial\ vertical\ velocity$$
$$g = acceleration\ of\ gravity = 32\frac{ft}{sec.^2} \approx 9.8\frac{m}{sec.^2}$$

Note that this only measures vertical height and disregards the horizontal position.

<u>Example</u>

A bullet is shot in the air off of a 128 foot building with an initial vertical velocity of 320 ft. per second.

a. Write a function to model the vertical height h as a function of time t

b. Set the equation equal to zero to find out how long it will take the bullet to hit the ground.

c. Suppose the horizontal velocity is 3000 ft. per second. Use the answer from part (b) to find the total horizontal distance the bullet will travel.

Distance/Rate/Time Problems

<u>Example</u> On a trip from Medford to Portland, John drove 275 at a certain average speed to make a meeting. After the meeting, he drove back to Medford 10 mph slower. The total time for the round trip was 8 hours. What was his average speed on each part of the trip?

Example
A boat travels 2 miles upriver (at a constant speed) drops off it's passengers, then travels 2 miles back. The time for the round trip was 3 hours. If the speed of the river is 3 mph, find the speed of the boat in still water.

Maximum and Minimum Problems

In the previous lessons, we learned that a parabola can have either a maximum or a minimum value. When

Example **Predicting Weather:**
The amount of precipitation in Sonoma, California, can be approximated by

$$P(x) = 0.2x^2 - 2.8x + 9.8$$

where $P(x)$ is in inches and x is the number of the month ($x = 1$ corresponds to January, etc.). In what month is there the least amount of precipitation, and how much precipitation occurs in that month?

Example **Business Model:**
When computing revenue (the amount of money that a business brings in) we often get a quadratic model based on a linear relationship between price and the number of units sold.

Suppose that the *Swedish Sister's* coffee stand would "sell" 2400 Lattes in a week if they gave them away for free (because that's all they can make). After collecting some data over several months, their marketing department has found that they sell 400 fewer Lattes for every dollar that they raise the price.

a) Write an equation for the number of lattes, n, as a function of the price p in dollars.

b) At what price will $n = 0$? Explain what this means for the Swedish Sisters?

c) Revenue is calculated by multiplying the number of lates sold, n, by the price p.
$$R = p \cdot n$$
Write an equation for revenue as a function of the price p by substituting your equation from part (a) for n.

d) According to this model, what is the maximum revenue that the coffee stand will make?

e) At what price will they reach this maximum?