9.5: Applications of Quadratic Equations

In this lesson, we will explore some real-world problems that involve quadratic equations.

When solving problems, break it up into steps like these:

1. Plan and Translate:
    - What information do we know? What do we need to find?
    - What mathematical properties, formulas, or techniques may be helpful?
    - Can you draw and label a picture or diagram for the situation?
2. Process
    - Define variables
    - Substitute known information into equations or formulas and solve equations
    - Use graphs to solve problem or guide your process.
3. Check
    - Substitute solution into formulas or equations to verify.
    - Use graphs to check solutions.

Number Problems

Example The product of two consecutive odd integers is 195. Find the integers.

Area Problems

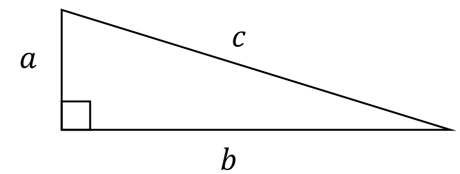
Start with a Triangle

An architect is designing the entryway of a restaurant. She wants to put a triangular window above the doorway. Due to energy restrictions, the window can only have an area of 120 square feet and the architect wants the base to be 4 feet more than twice the height. Find the base and height of the window.

Pythagorean Theorem

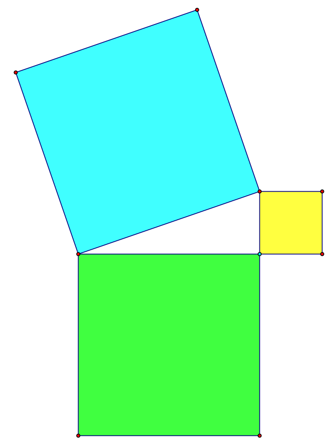
In this lesson we will investigate several applications of radical expressions and equations beginning with one of the most important equations in math, the Pythagorean Theorem.

I. Right Triangles and The Pythagorean Theorem



Pythagorean Theorem: *"In all right triangles, The sum of the area of the squares on the legs is equal to the area of the square on the hypotenuse."*

Algebraically: *If a triangle is a right triangle with legs of length a and b, and a hypotenuse of length c, then…*



$$a^2 + b^2 = c^2$$

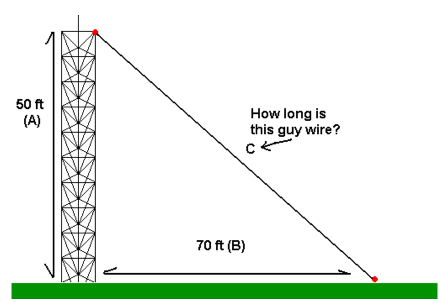
Example 1 Find the missing side for the following right triangles. State your answer in simplest radical form.

a) $a = 10, c = 26$

b) The shortest leg of a right triangle is 2 cm shorter than the other leg. The hypotenuse is 10. What are the lengths of the two legs of the triangle?

Example 2

a) Suppose a 50 foot **cell tower** is being built, and a guy wire is going to be attached to the ground 70 feet from the base. How long will this wire need to be?



b) Suppose we have a 100 foot **wire** that we want to use, how far from the base should the anchor be put in the ground to attach this 100 ft. guy wire?

Falling Objects

The distance that a free-falling object travels is $s(t) = 16t^2$. If you drop a penny from the top of the roof of the Empire state building at 1250 feet high, how long will it take to reach the ground?

Projectiles

The height of a projectile (an object that is propelled into the air without any means of self-propulsion) is determined by the function

$$s(t) = s_0 + v_0 t - \frac{1}{2}gt^2$$

Where

$$s_0 = intial\ height,\ \ v_0 = initial\ vertical\ velocity$$

$$g = acceleration\ of\ gravity = 32\frac{ft}{sec.^2} \approx 9.8\frac{m}{sec.^2}$$

Note that this only measures vertical height and disregards the horizontal position.

Example

A bullet is shot in the air off of a 128 foot building with an initial vertical velocity of 320 ft. per second.

a. Write a function to model the vertical height h as a function of time t

b. Set the equation equal to zero to find out how long it will take the bullet to hit the ground.

c. Suppose the horizontal velocity is 3000 ft. per second. Use the answer from part (b) to find the total horizontal distance the bullet will travel.

Maximum and Minimum Problems

In the previous lessons, we learned that a parabola can have either a maximum or a minimum value. When

Example **Predicting Weather:**

The amount of precipitation in Sonoma, California, can be approximated by

$$P(x) = 0.2x^2 - 2.8x + 9.8$$

where $P(x)$ is in inches and x is the number of the month ($x = 1$ corresponds to January, etc.). In what month is there the least amount of precipitation, and how much precipitation occurs in that month?

Try a Rectangle Problem!

Old MacDonald had a farm…. On that farm he had some pigs…. and wants to make a rectangular pen for his pigs against his barn, so he needs to fence in *3 sides of the pen*. He has 80 feet of fencing and he wants to make a pen with the largest amount of area for his pigs.

What is the largest area that he can enclose?

What dimensions give him this area?

Finding a Regression Model

When analyzing data, we can use technology to find a function that closely approximates the collected values. This process of going from specific points to an equation is called **regression**. On the next page are the steps for using a TI-83 or TI-84 calculator to create a scatterplot and find a ***regression equation*** for a set of data.

Steps for finding a regression equation:

Enter Data:

1. Go to STAT and select 1 to go to EDIT.
2. Enter the x-values into L_1 and enter the y-values into L_2.

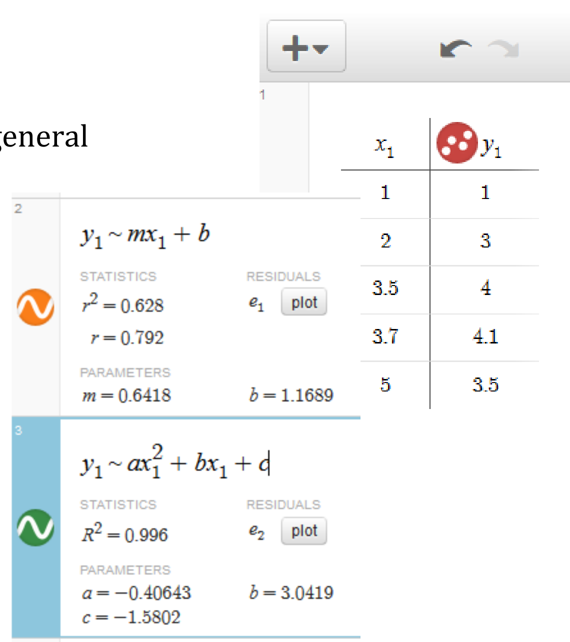
Creating a scatterplot:

3. Now we need to graph these values in a scatterplot with the following steps.
4. Press 2nd Y= to get the STAT PLOT menu.
5. Choose option 1 to get Plot 1
6. Select [On] to turn the plot on.
7. Choose the scatterplot option.
8. Since we used the lists L_1 and L_2., we do not need to change the Xlist or Ylist. If we did use a different list, we would need to change these.
9. To view the graph, hit the ZOOM button and choose ZoomStat which is option 9.

Finding a regression equation:

10. Now if we want find a linear regression, press STAT and choose CALC.
11. Choose the type of regression you want. For now, we will use LinReg and QuadReg for linear and quadratic regression.
12. This will send you back to the homescreen. You now just need to hit enter to get your equation.
13. (optional) If you want to store the equation in the equation editor, do this before pressing enter: Type a left-parenthesis, go to [Vars]→[Y-Vars] and select [Y1], type a right – parenthesis, then hit enter. Hit the Y= button to see your equation stored.

Regression using Desmos.com



1. Make a table and enter data
2. Use the statistics approximation function ~ to type the general regression forms (don't forget the subscript "1" after the variables):
   Linear: $y_1 \sim mx_1 + b$
   Quadratic: $y_1 \sim ax_1^2 + bx_1 + c$
3. To extrapolate another value using this function, write the function with a number in place of x. If $x = 25$, type
   Linear: $m(25) + b$
   Quadratic: $a(25)^2 + b(25) + c$
   Then compute a specific value like $f(25)$

Example **Radar Gun Simulation**
Roll a ball down a ramp at different intervals and measure the time. Then use regression to find an equation to model the data and make a predition.

a) Use regression to find a quadratic function that fits the data.

Distance down the ramp	Time, t, in seconds
30	
40	
50	
60	
70	
80	
90	

b) Use the function to predict how long it would take the ball to roll down a 200cm ramp at the same angle.

c) Use the model to predict how long a ramp you would need for the ball to take 60 seconds to roll the entire length.

Example A rocket was shot in the air and the height measured at several times.

a) Use the data to calculate a quadratic regression equation for the height h, in feet, as a function of time t, in seconds.

Time (sec)	0	1	2	3
Height (ft)	0	460	925	1300

b) Use the function to approximate the maximum height of the rocket.

c) Use the function to approximate when the rocket will hit the ground.