9-6: Graphing Quadratics using Properties

In this lesson, we will move from quadratic *equations* to quadratic *functions.* The key difference between these two is that a function has two variables, one input (*independent*) variable, and one output (*dependent*) variable. Our primary interest at this point is the graph of the quadratic function

$$f(x) = ax^2 + bx + c$$

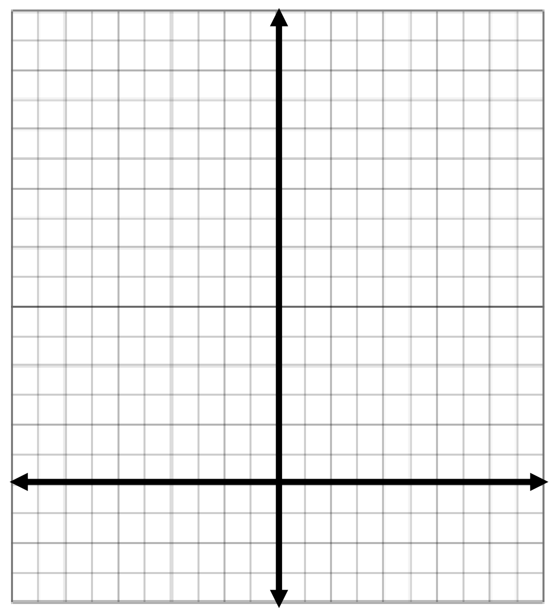
which will always be a U-shaped graph called a *parabola.*

The Graph of $f(x) = ax^2$

We begin with the simplest form of a quadratic function $f(x) = ax^2$.

<u>*Explore*</u> Complete the table of values for $f(x) = x^2$ and use it to graph the function.

x	$f(x) = x^2$
-3	
-2	
-1	
0	
1	
2	
3	



The shape of this graph is called a ***parabola***. The lowest point on the graph is called the ***vertex***. The vertical line that divides the parabola into two symmetrical halves is the ***axis of symmetry.***

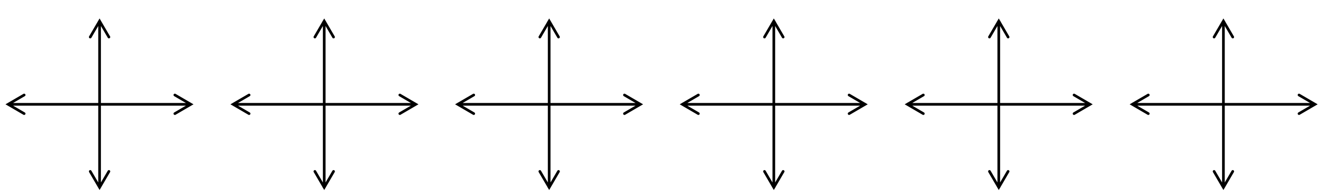
<u>***Consider this:***</u>

- What are the coordinates of the vertex of $f(x) = x^2$?
- What is the equation of the axis of symmetry for $f(x) = x^2$?

<u>*Explore more*</u> How does the graph of $f(x) = ax^2$ change as the value of a changes.

Use your graphing calculator to graph $y_1 = x^2$. Then graph each of the following in y_2 .

$y_2 = 3x^2$ $\quad$ $y_2 = .8x^2$ $\quad$ $y_2 = .2x^2$ $\quad$ $y_2 = -x^2$ $\quad$ $y = -3x^2$ $\quad$ $y = -.2x^2$



In the graph of $y = ax^2$, how does the value of a change the shape or position of the parabola relative to the graph of $y = x^2$?

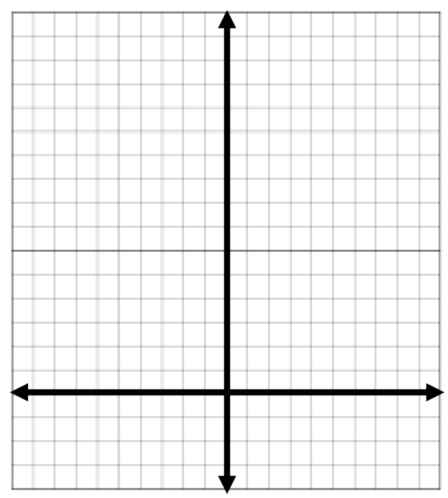
- If $a > 0$, then the graph opens ________________________

- If $a < 0$, then the graph opens ________________________

- If $0 < a < 1$, or $-1 < a < 0$, then shape of the graph is ___________________________ or is "stretched" vertically.

- If $a < -1$, or $a > 1$, then the shape of the graph is ______________________________ or is "shrunk" vertically.

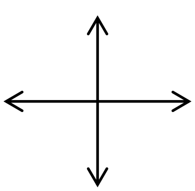
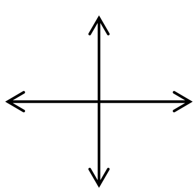
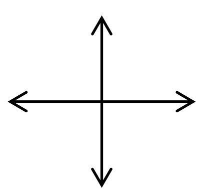
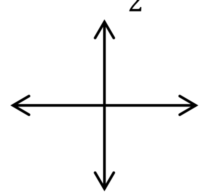
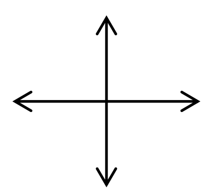
Example Use your observations to graph $y = \frac{1}{2}x^2$.
Describe how this graph is different than $y = x^2$.



The Graph of $f(x) = a(x-h)^2$

Multiplying by a constant, a, changes the shape but it does not move the graph of the parabola. Let's investigate some moving parabolas.

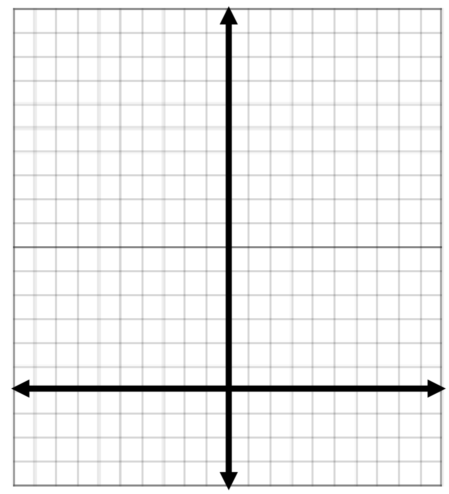
Keep Exploring. How does the graph of $f(x) = a(x-h)^2$ change as the value of h changes.
Use your graphing calculator to graph each of the following pairs of functions.

$y_1 = x^2$ $y_2 = (x-3)^2$	$y_1 = x^2$ $y_2 = (x+3)^2$	$y_1 = 2x^2$ $y_2 = 2(x-4)^2$	$y_1 = -\frac{1}{2}x^2$ $y_2 = -\frac{1}{2}(x+4)^2$	$y_1 = -3x^2$ $y_2 = -3(x-5)^2$
				

In the graph of $y = a(x+h)^2$, how does the value of h change the shape or position of the parabola relative to the graph of $y = ax^2$?

- If $h > 0$, then the graph moves ________________________
- If $h < 0$, then the graph moves ________________________

Example Use your observations to graph $y = (x+3)^2$.
Describe how this graph is different than $y = x^2$.



The Graph of $f(x) = a(x-h)^2 + k$

In the previous investigation, we saw how adding an number to the function *before* squaring moves the parabola left or right. Now let's consider what happens when we add a number to the function *after* squaring.

Consider This: The value of a function, $f(x)$, for a certain value of x is just the height of the point on the graph.

So, what would happen to the graph if we take the function $f(x)$ and change it to $f(x) + 2$? That is, what if we change $f(x) = x^2$ to $g(x) = x^2 + 2$?

Likewise, what happens to the graph if we change $f(x) = x^2$ to $g(x) = x^2 - 2$?

→ If we add a *positive* constant k to get $f(x)$, the graph moves _________________

→If we add a *negative* constant k from $f(x)$, the graph moves __________________

This affects the greatest value (called a **maximum**) and the least value (called the **minimum**) of the function.

Example Find the maximum or the minimum (y) value of each of these quadratic functions

a. $y = x^2 + 5$ b. $f(x) = -3x^2 - 4$ c. $g(x) = 2(x-5)^2 + 7$

Finding the vertex from standard form

Suppose we have a function in standard form $f(x) = ax^2 + bx + c$. If we complete the square, this standard form equation changes into

$$f(x) = a\left[x - \left(-\frac{b}{2a}\right)\right]^2 + c$$

Finding the vertex in standard form:
The general vertex form shows us that for any quadratic function in the form $f(x) = ax^2 + bx + c$ ➢ The x-coordinate of the vertex is at $-\boldsymbol{\frac{b}{2a}}$, ➢ the **axis of symmetry** is at $x = -\frac{b}{2a}$, and ➢ the **vertex** is at $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$ ➢ the maximum or minimum (depending on a) is at $f\left(-\frac{b}{2a}\right)$

Remember: *The axis of symmetry always passes through the <u>vertex</u>!*

<u>Example</u>. Find the axis of symmetry, the coordinates of the vertex, the minimum or maximum value, and sketch a *rough* graph for each function.

a) $f(x) = -2x^2 + 3x - 5$

b) $g(x) = 5x^2 + x - 22$

Determining the Intercepts

The intercepts of a function are key values that often need to be found. Since the intercepts are on the axes, we know that one of the coordinates must be zero. We find these intercepts as follows:

x-intercepts: when $y = f(x) = 0$, **y-intercept: when $x = 0$**

<u>Example</u>: Find the x-intercepts and y-intercepts of these functions.

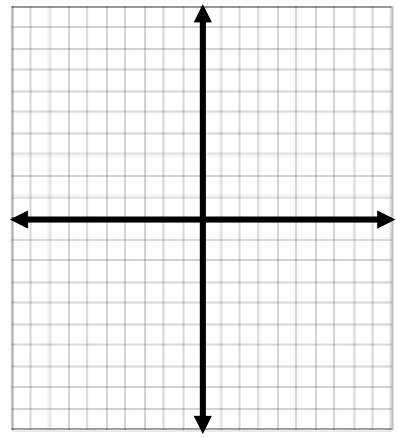
a) $f(x) = -2x^2 + 3x - 5$

b) $g(x) = 5x^2 + x - 22$

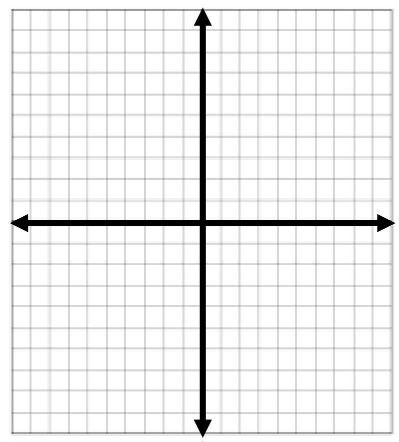
We can now graph quadratic function in ***standard form*** ($y = ax^2 + bx + c$) by finding the vertex, finding easy intercepts, then using the a value to find several more points and graph.

Example Find the vertex and axis of symmetry of each function and sketch a graph.

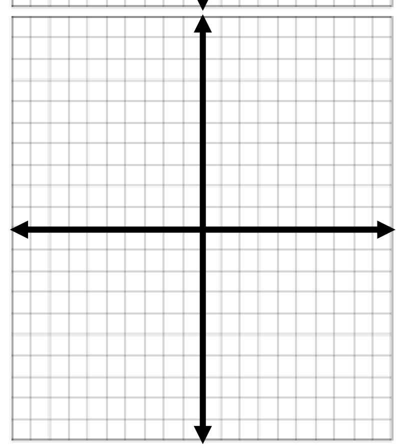
a) $y = x^2 + 2x - 3$



b) $g(x) = 2\,x^2 - 8\,x + 9$



c) $h(x) = -2x^2 + 10x - 7$



Example

The height of a certain ball that is thrown off of a 20 ft. building can be calculated using the function

$$h(t) = 20 + 10t - 16t^2$$

for the height $h(t)$, in feet, at time t, in seconds. Find the maximum height of the ball, then find the time when the ball will hit the ground.