9-7: Graphing Quadratics using Transformations

Transforming the function $f(x) = x^2$

A *transformation* of a graph is a change that makes a graph slide, flip, or stretch on the coordinate plane. We have seen that the graph of $y = ax^2$ takes the graph of $y = x^2$ and stretches it, or flips it across the x-axis. Now, let's look at some other numbers that can change the position of shape of the graph.

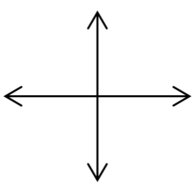
The Graph of $f(x) = a(x - h)^2$

Multiplying by a constant, a, changes the shape but it does not move the graph of the parabola. Let's investigate some moving parabolas.

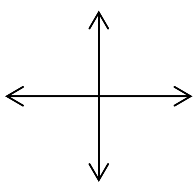
<u>*Keep Exploring*</u>. How does the graph of $f(x) = a(x - h)^2$ change as the value of h changes.

Use your graphing calculator to graph each of the following pairs of functions.

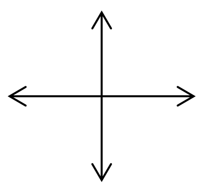
$y_1 = x^2$
$y_2 = (x - 3)^2$



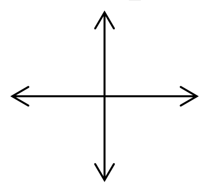
$y_1 = x^2$
$y_2 = (x + 3)^2$



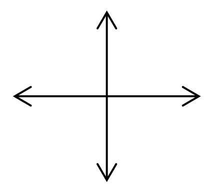
$y_1 = 2x^2$
$y_2 = 2(x - 4)^2$



$y_1 = -\frac{1}{2}x^2$
$y_2 = -\frac{1}{2}(x + 4)^2$



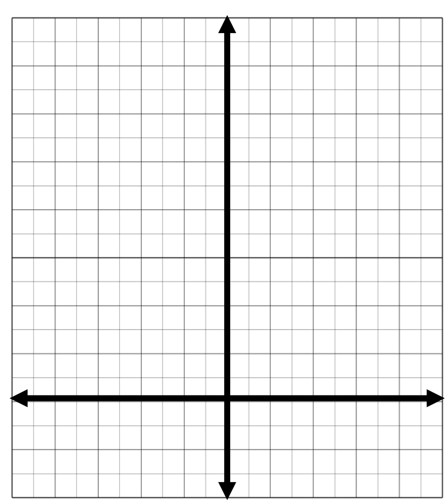
$y_1 = -3x^2$
$y_2 = -3(x - 5)^2$



In the graph of $y = a(x + h)^2$, how does the value of h change the shape or position of the parabola relative to the graph of $y = ax^2$?

- If $h > 0$, then the graph moves ________________
- If $h < 0$, then the graph moves ________________

<u>*Example*</u> Use your observations to graph $y = (x + 3)^2$.
Describe how this graph is different than $y = x^2$.



Step 1: Find the vertex

Step 2: Use the "a" value of 1 (it's invisible) to graph the shape of the graph $y = 1x^2$ at the new vertex.

The Graph of $f(x) = a(x-h)^2 + k$

In the previous investigation, we saw how adding an number to the function *before* squaring moves the parabola left or right. Now let's consider what happens when we add a number to the function *after* squaring.

Consider This: The value of a function, $f(x)$, for a certain value of x is just the height of the point on the graph.

So, what would happen to the graph if we take the function $f(x)$ and change it to $f(x) + 2$? That is, what if we change $f(x) = x^2$ to $g(x) = x^2 + 2$?

Likewise, what happens to the graph if we change $f(x) = x^2$ to $g(x) = x^2 - 2$?

→ If we add a *positive* constant k to get $f(x)$, the graph moves _______________

→If we add a *negative* constant k from $f(x)$, the graph moves ________________

This affects the greatest value (called a **maximum**) and the least value (called the **minimum**) of the function.

Example Find the maximum or the minimum (y) value of each of these quadratic functions

a. $y = x^2 + 5$

b. $f(x) = -3x^2 - 4$

c. $g(x) = 2(x-5)^2 + 7$

Vertex Form: A Useful Form

By observing transformations of the graph of the quadratic equation $y = x^2$ we have learned how to move the parabola vertically or horizontally, flip it vertically, and modify its rate of change (i.e. its vertical stretch). These transformations can all be summarized in the general form, $f(x) = a(x-h)^2 + k$, which we call vertex form. Below is a description of the effects of the parameters a, h, and k.

Vertex form of a quadratic function: $f(x) = a(x-h)^2 + k$

For constants a, h, and k, the shape of the graph can be determined by the following:

a: Affects the vertical stretch ($|a| > 1$)or shrink ($|a| < 1$) of the graph.
Determines if the graph opens up ($a > 0$) and has a *minimum*,
or if it opens down ($a < 0$) and has a *maximum*.

h: Shifts the graph horizontally h units.
Remember…$(h-3)^2$ moves *right* and $(h+3)^2$ moves *left*!
Axis of symmetry is $x = h$

k: Shifts the graph vertically k units.
Maximum or **minimum** value (determined by value of a) equals k.

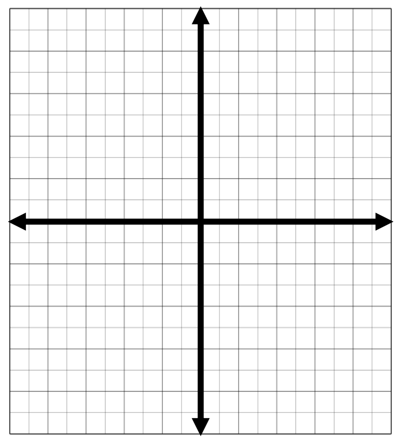
Vertex: The coordinates of the **vertex** are (h, k).

Try These:

1. Find the equation of the parabola that is twice as steep as $y = x^2$ and is moved left 5 units and down 14 units.

2. Describe how the graph of $y = -(x - 18)^2 + 9$ differs from the graph of $y = x^2$, and find the coordinates of its vertex.

Example For each function, sketch the graph and find the coordinates of the vertex, the axis of symmetry, the maximum or minimum value, and find the domain and range.



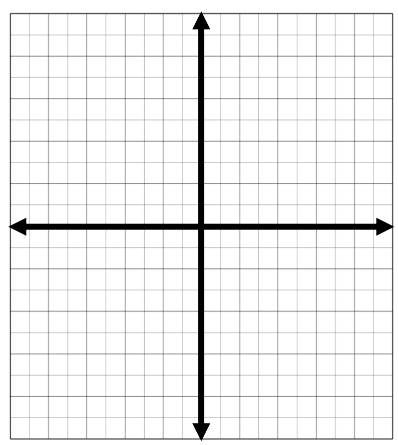
a) $f(x) = \frac{1}{2}(x - 2)^2 - 4$

vertex: axis of symmetry:

max or min:

Domain: Range:

b) $g(x) = -2(x + 1)^2 + 2$



vertex: axis of symmetry:

max or min:

Domain: Range:

Consider this: Observe that this vertex form contains $(x-h)^2$. What technique have we learned that changes a quadratic in the form ax^2+bx+c in to a binomial squared?.... It's completing the square.

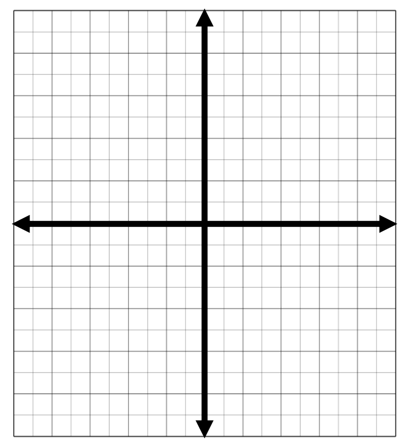
Example Use completing the square to find the vertex of

$$g(x)=x^2+2x-6$$

We can now change a quadratic function from ***standard form*** ($y=ax^2+bx+c$) into *vertex form.* This can be very useful to understand a function better.

Example Find the vertex and axis of symmetry of each function and sketch a graph.

a) $y=x^2+2x-3$



b) $g(x)=2\,x^2-8\,x+9$