9-7: Graphing Quadratics using Transformations

Vertex Form: A Useful Form

By observing transformations of the graph of the quadratic equation $y = x^2$ we have learned how to move the parabola vertically or horizontally, flip it vertically, and modify its rate of change (i.e. its vertical stretch). These transformations can all be summarized in the general form, $f(x) = a(x-h)^2 + k$, which we call vertex form. Below is a description of the effects of the parameters a, h, and k.

Vertex form of a quadratic function: $\boldsymbol{f(x) = a(x-h)^2 + k}$

For constants a, h, and k, the shape of the graph can be determined by the following:

a: Affects the vertical stretch ($|a| > 1$)or shrink ($|a| < 1$) of the graph.
Determines if the graph opens up ($a > 0$) and has a *minimum*,
or if it opens down ($a < 0$) and has a *maximum*.

h: Shifts the graph horizontally h units.
Remember…$(h-3)^2$ moves *right* and $(h+3)^2$ moves *left*!
Axis of symmetry is $x = h$

k: Shifts the graph vertically k units.
Maximum or **minimum** value (determined by value of a) equals k.

Vertex: The coordinates of the **vertex** are (h, k).

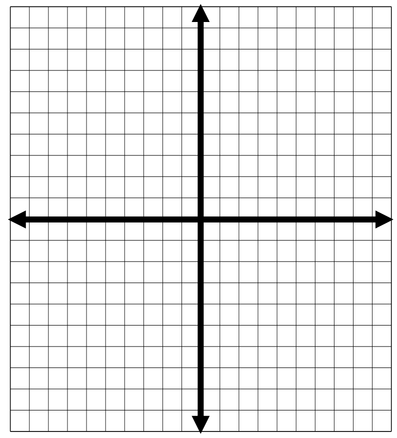
Try These:

1. Find the equation of the parabola that is twice as steep as $y = x^2$ and is moved left 5 units and down 14 units.

2. Describe how the graph of $y = -(x-18)^2 + 9$ differs from the graph of $y = x^2$, and find the coordinates of its vertex.

Example For each function, sketch the graph and find the coordinates of the vertex, the axis of symmetry, the maximum or minimum value, and find the domain and range.

a) $f(x) = (x - 2)^2 - 4$

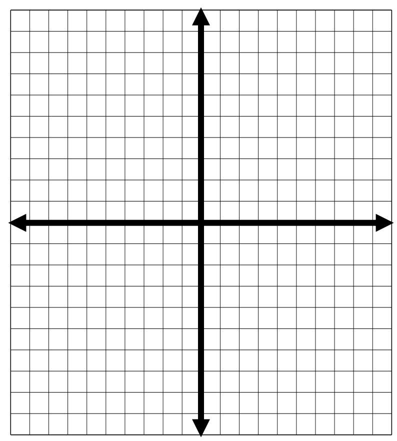


vertex: axis of symmetry:

max or min:

Domain: Range:

b) $g(x) = \frac{1}{2}(x + 1)^2 + 2$

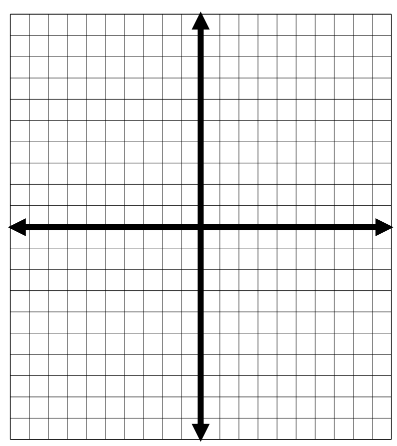


vertex: axis of symmetry:

max or min:

Domain: Range:

c) $h(x) = -(x + 3)^2$



vertex: axis of symmetry:

max or min:

Domain: Range:

Consider this: Observe that this vertex form contains $(x-h)^2$. What technique have we learned that changes a quadratic in the form ax^2+bx+c in to a binomial squared?.... It's completing the square.

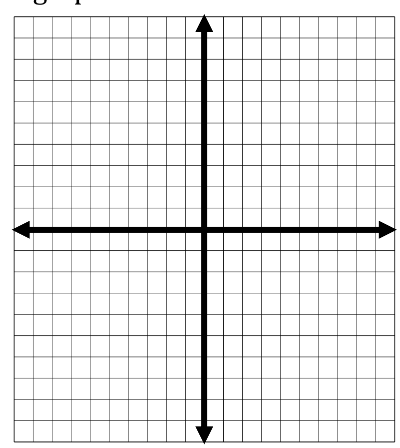
Example Use completing the square to find the vertex of

$$g(x)=x^2+2x-6$$

We can now change a quadratic function from ***standard form*** $(y=ax^2+bx+c)$ into *vertex form.* This can be very useful to understand a function better.

Example Find the vertex and axis of symmetry of each function and sketch a graph.

a) $y=x^2+2x-3$



b) $g(x)=2\,x^2-8\,x+9$