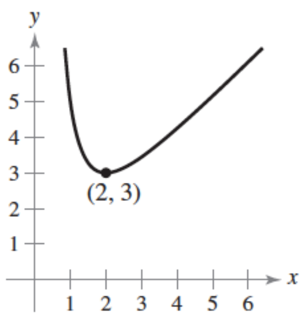
Calculus Name: Date:

3A Exercises

Extrema on an Interval

3. $g(x) = x + \frac{4}{x^2}$

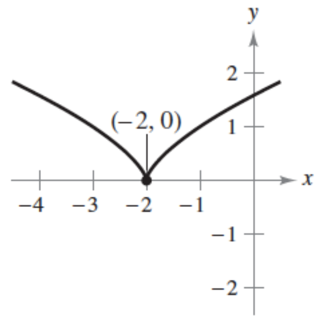


$$f(x) = x + \frac{4}{x^2} = x + 4x^{-2}$$

$$f'(x) = 1 - 8x^{-3} = 1 - \frac{8}{x^3}$$

$$f'(2) = 0$$

5. $f(x) = (x + 2)^{2/3}$



$$f(x) = (x + 2)^{2/3}$$

$$f'(x) = \frac{2}{3}(x + 2)^{-1/3}$$

$f'(-2)$ is undefined.

Find the any critical numbers of the function.

11. $f(x) = x^3 - 3x^2$

$$f(x) = x^3 - 3x^2$$

$$f'(x) = 3x^2 - 6x = 3x(x - 2)$$

Critical numbers: $x = 0, 2$

15. $h(x) = \sin^2 x + \cos x$

$0 < x < 2\pi$

$$h(x) = \sin^2 x + \cos x, \quad 0 < x < 2\pi$$

$$h'(x) = 2 \sin x \cos x - \sin x = \sin x(2 \cos x - 1)$$

Critical numbers in $(0, 2\pi)$: $x = \frac{\pi}{3}, \pi, \frac{5\pi}{3}$

14. $f(x) = \frac{4x}{x^2 + 1}$

$$f'(x) = \frac{4(x^2 + 1) - 4x(2x)}{(x^2 + 1)^2} = \frac{-4x^2 + 4}{(x^2 + 1)^2} = 0$$

$-4x^2 + 4 = 0$

$x^2 = 1, \qquad x = \pm 1$

Critical numbers: $x = -1, 1$

Find the absolute extrema of the function on the interval

19. $g(x) = x^2 - 2x,\ [0, 4]$

$g(x) = x^2 - 2x, \quad [0, 4]$

$g'(x) = 2x - 2 = 2(x - 1)$

Critical number: $x = 1$

Left endpoint: $(0, 0)$

Critical number: $(1, -1)$ Minimum

Right endpoint: $(4, 8)$ Maximum

25. $g(t) = \frac{t^2}{t^2 + 3},\ [-1, 1]$

$g(t) = \frac{t^2}{t^2 + 3}, \quad [-1, 1]$

$g'(t) = \frac{6t}{(t^2 + 3)^2}$

Left endpoint: $\left(-1, \frac{1}{4}\right)$ Maximum

Critical number: $(0, 0)$ Minimum

Right endpoint: $\left(1, \frac{1}{4}\right)$ Maximum

23. $y = 3x^{2/3} - 2x,\ [-1, 1]$

$f(x) = 3x^{2/3} - 2x, \quad [-1, 1]$

$f'(x) = 2x^{-1/3} - 2 = \frac{2(1 - \sqrt[3]{x})}{\sqrt[3]{x}}$

Left endpoint: $(-1, 5)$ Maximum

Critical number: $(0, 0)$ Minimum

Right endpoint: $(1, 1)$

33. $f(x) = \cos \pi x,\ \left[0, \frac{1}{6}\right]$

$f(x) = \cos \pi x, \left[0, \frac{1}{6}\right]$

$f'(x) = -\pi \sin \pi x$

Left endpoint: $(0, 1)$ Maximum

Right endpoint: $\left(\frac{1}{6}, \frac{\sqrt{3}}{2}\right)$ Minimum

36. $y = \tan\left(\frac{\pi x}{8}\right),\ [0, 2]$

$$\frac{dy}{dx} = \frac{\pi}{8}\sec^2\left(\frac{\pi}{8}x\right) = 0$$

$$\sec^2\left(\frac{\pi}{8}x\right) = 0$$

There is no solution to this equation because $\sec^2\theta \geq 1$ for all values of θ. So, there are no critical points.

we test the endpoints:

at $x = 0,\ y = \tan(0) = 0$

at $x = 2, y = \tan\left(\frac{\pi}{4}\right) = 1$

Minimum at (0,0)

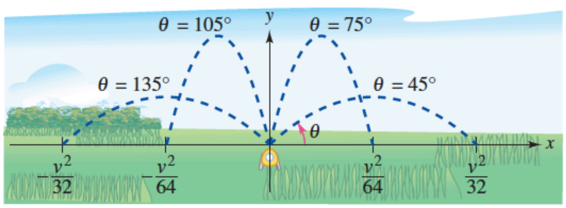
Maximum at (2,1)

Choose one of the following applications to complete

62. *Lawn Sprinkler* A lawn sprinkler is constructed in such a way that $d\theta/dt$ is constant, where θ ranges between 45° and 135° (see figure). The distance the water travels horizontally is

$$x = \frac{v^2 \sin 2\theta}{32}, \quad 45° \le \theta \le 135°$$

where v is the speed of the water. Find dx/dt and explain why this lawn sprinkler does not water evenly. What part of the lawn receives the most water?



Water sprinkler: $45° \le \theta \le 135°$

■ **FOR FURTHER INFORMATION** For more information on the "calculus of lawn sprinklers," see the article "Design of an Oscillating Sprinkler" by Bart Braden in *Mathematics Magazine*. To view this article, go to the website *www.matharticles.com*.

Find the critical points of $dx/d\theta$

$$\frac{dx}{d\theta} = \frac{v^2}{32}(2\cos 2\theta) = \frac{v^2}{16}\cos 2\theta$$

$$0 = \frac{v^2}{16}\cos 2\theta$$

$$0 = \cos 2\theta$$

$$2\theta = 90°, \quad or \quad 2\theta = 270°$$

$$\theta = 45°, \quad or \quad \theta = 135°$$

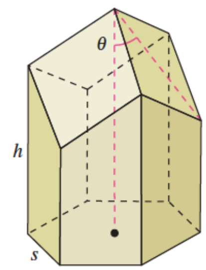
This means that there is a critical point at these two values, which happen to be the endpoints of the interval.

At these two end points, the instantaneous change (i.e. the derivative $\frac{dx}{d\theta}$) in the sprinkler movement will be a minimum, meaning it will be moving the slowest here. So, the amount of water that will land at the extremities will be a maximum because the change in the angle has stopped briefly.

63. *Honeycomb* The surface area of a cell in a honeycomb is

$$S = 6hs + \frac{3s^2}{2}\left(\frac{\sqrt{3} - \cos\theta}{\sin\theta}\right)$$

where h and s are positive constants and θ is the angle at which the upper faces meet the altitude of the cell (see figure). Find the angle θ $(\pi/6 \le \theta \le \pi/2)$ that minimizes the surface area S.



Find the Critical points of $dS/d\theta$

$$\frac{dS}{d\theta} = \frac{3s^2}{2}\left(\frac{\sin^2\theta - \cos\theta\left(\sqrt{3} - \cos\theta\right)}{\sin^2\theta}\right) = 0$$

$$0 = \sin^2\theta - \cos\theta\left(\sqrt{3} - \cos\theta\right)$$

$$0 = \sin^2\theta - \sqrt{3}\cos\theta + \cos^2\theta$$

$$0 = 1 - \sqrt{3}\cos\theta$$

$$\frac{1}{\sqrt{3}} = \cos\theta$$

$$\theta \approx .955\ radians$$

Check $S(.955), S\left(\frac{\pi}{6}\right), S\left(\frac{\pi}{2}\right)$

And we find that $S(.955)$ is a minimum.