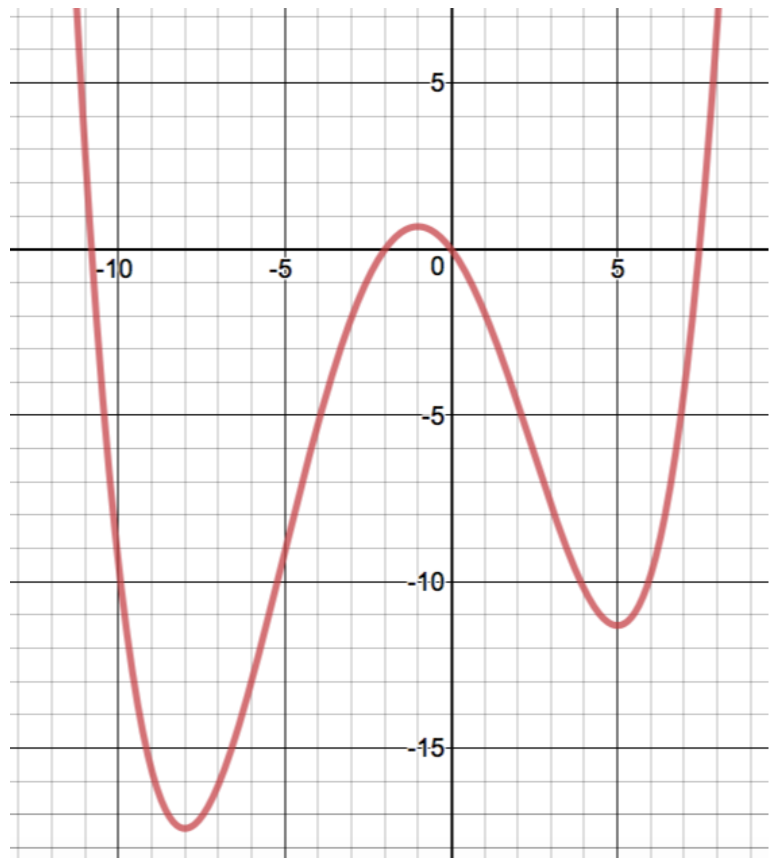
3A: Extrema on an Interval

In this lesson, we will see how we can use derivatives to help us find very important extrema of a function on a certain interval.

A Walk Down Memory Lane…

Let's see what we remember from Pre-Calculus. Use the graph to the right to approximate the following.



a) Where is the absolute (a.k.a. global) minimum?

b) What is the absolute minimum?

c) Where is a relative (a.k.a. local) minimum?

d) What is the relative (a.k.a. local) minimum?

e) Where is the absolute maximum?

f) What is the absolute maximum?

g) What is the absolute maximum of the interval $[-8, -2]$

THEOREM 3.1 THE EXTREME VALUE THEOREM

If f is continuous on a closed interval $[a, b]$, then f has both a minimum and a maximum on the interval.

h) Write a possible equation for this polynomial function in factored form.

i) What is the value of the derivative at these extrema?

Critical Points

Now we can see that if there is an extrema at x_0 , then $f'(x_0) = 0$.

So, is the converse true?

That is, if the $f'(x_0) = 0$, is there an extrema at x_0?

DEFINITION OF EXTREMA

Let f be defined on an interval I containing c.

1. $f(c)$ is the **minimum of f on I** if $f(c) \le f(x)$ for all x in I.
2. $f(c)$ is the **maximum of f on I** if $f(c) \ge f(x)$ for all x in I.

The minimum and maximum of a function on an interval are the **extreme values,** or **extrema** (the singular form of extrema is extremum), of the function on the interval. The minimum and maximum of a function on an interval are also called the **absolute minimum** and **absolute maximum,** or the **global minimum** and **global maximum,** on the interval.

__Consider this__: Find the values that make the derivative zero for $f(x) = x^3$.

These values of c that make $f'(c) = 0$ sometimes give us an extrema, but they are always important, so we call these **critical numbers**.

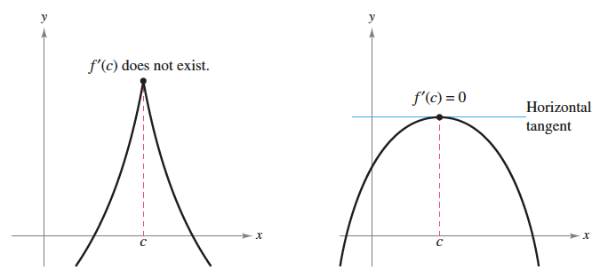
DEFINITION OF A CRITICAL NUMBER

Let f be defined at c. If $f'(c) = 0$ or if f is not differentiable at c, then c is a **critical number** of f.

As we saw above

Relative Extrema Theorem

If $f(x)$ has a minimum or maximum at $x = c$, then c is a critical point.



c is a critical number of f.

However, __the converse is not true__.

A critical point __*does not*__ guarantee an extrema!

GUIDELINES FOR FINDING EXTREMA ON A CLOSED INTERVAL

To find the extrema of a continuous function f on a closed interval $[a, b]$, use the following steps.

1. Find the critical numbers of f in (a, b).
2. Evaluate f at each critical number in (a, b).
3. Evaluate f at each endpoint of $[a, b]$.
4. The least of these values is the minimum. The greatest is the maximum.

Now, let's apply these steps to the problems on your assignment.

Example **Find the critical points of these functions on the given intervals**

a) $f(x) = x^2 - 2x$ on $[0,3]$

b) $g(x) = \sin x$ on $\left[\frac{0,3\pi}{2}\right]$