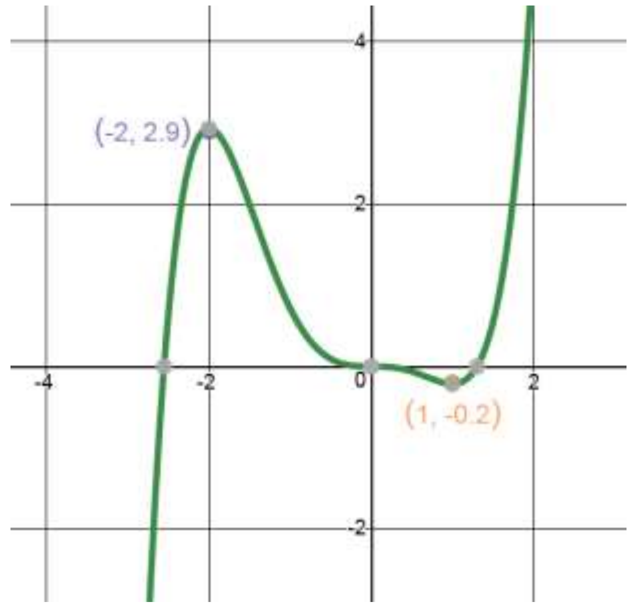
Name: Date:

3B: Intervals of Increasing and Decreasing

In the last lesson, we saw that when $f'(x) = 0$ or $f'(x)$ does not exist, we get critical points. These critical points are often (but not always) extrema. Now what we want to investigate is what happens in between these extrema.



Derivatives not at Extrema

Consider the graph of $f(x) = -\frac{2}{3}x^3 + \frac{1}{4}x^4 + \frac{1}{5}x^5$

1. Where are the critical points?

2. For each interval, decide if the function is increasing or decreasing, and make a generalization about the derivative for all points on the interval.

Interval	$(-\infty, -2)$	$(-2,0)$	$(0,1)$	$(1, \infty)$
Increasing/Decreasing?				
Derivative				

What can we conclude?

If $f(x)$ is increasing, then $f'(x)$ is ______________________________

If $f(x)$ is decreasing, then $f'(x)$ is ______________________________

If $f(x)$ is constant, then $f'(x)$ is ______________________________

THEOREM 3.5 TEST FOR INCREASING AND DECREASING FUNCTIONS

Let f be a function that is continuous on the closed interval $[a, b]$ and differentiable on the open interval (a, b).

1. If $f'(x) > 0$ for all x in (a, b), then f is increasing on $[a, b]$.
2. If $f'(x) < 0$ for all x in (a, b), then f is decreasing on $[a, b]$.
3. If $f'(x) = 0$ for all x in (a, b), then f is constant on $[a, b]$.

Try it Out:

1. Determine the intervals of increasing and decreasing for $f(x) = x^3 - 3x^2 - 24x + 1$

2. Determine the intervals of increasing and decreasing for $f(x) = (x^2 - 4)^{2/3}$

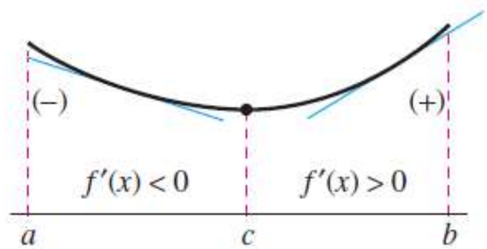
3. Determine the intervals of increasing and decreasing for $f(x) = \frac{x^4+1}{x^2}$

Can number lines tell us more than increasing and decreasing? Look at the examples above and determine what type of extrema each is.

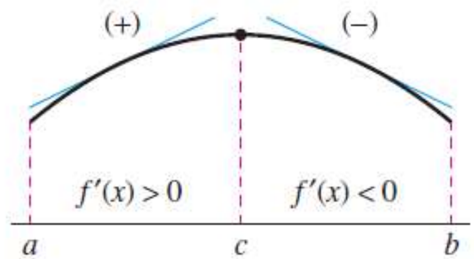
THEOREM 3.6 THE FIRST DERIVATIVE TEST

Let c be a critical number of a function f that is continuous on an open interval I containing c. If f is differentiable on the interval, except possibly at c, then $f(c)$ can be classified as follows.

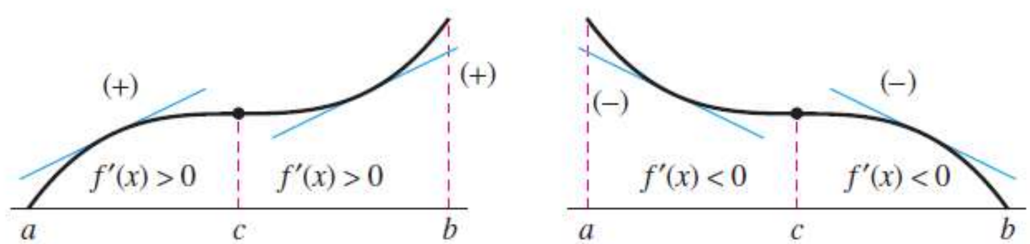
1. If $f'(x)$ changes from negative to positive at c, then f has a *relative minimum* at $(c, f(c))$.
2. If $f'(x)$ changes from positive to negative at c, then f has a *relative maximum* at $(c, f(c))$.
3. If $f'(x)$ is positive on both sides of c or negative on both sides of c, then $f(c)$ is neither a relative minimum nor a relative maximum.



Relative minimum



Relative maximum



Neither relative minimum nor relative maximum