# 3C.1 Exercises

## Concavity and The 2nd Derivative

Find the Points of Inflection and the Intervals of Concavity

**19.** $f(x) = \frac{1}{2}x^4 + 2x^3$

$f'(x) = 2x^3 + 6x^2$

$f''(x) = 6x^2 + 12x = 6x(x + 2)$

$f''(x) = 0$ when $x = 0, -2$

Concave upward: $(-\infty, -2), (0, \infty)$

Concave downward: $(-2, 0)$

Points of inflection: $(-2, -8)$ and $(0, 0)$

**21.** $f(x) = x^3 - 6x^2 + 12x$

$f'(x) = 3x^2 - 12x + 12$

$f''(x) = 6(x - 2) = 0$ when $x = 2$.

Concave upward: $(2, \infty)$

Concave downward: $(-\infty, 2)$

Point of inflection: $(2, 8)$

**23.** $f(x) = \frac{1}{4}x^4 - 2x^2$

$f'(x) = x^3 - 4x$

$f''(x) = 3x^2 - 4$

$f''(x) = 3x^2 - 4 = 0$ when $x = \pm\frac{2}{\sqrt{3}}$.

Points of inflection: $\left(\pm\frac{2}{\sqrt{3}}, -\frac{20}{9}\right)$

| Test interval: | $-\infty < x < -\frac{2}{\sqrt{3}}$ | $-\frac{2}{\sqrt{3}} < x < \frac{2}{\sqrt{3}}$ | $\frac{2}{\sqrt{3}} < x < \infty$ |
|---|---|---|---|
| Sign of $f''(x)$: | $f''(x) > 0$ | $f''(x) < 0$ | $f''(x) > 0$ |
| Conclusion: | Concave upward | Concave downward | Concave upward |

**25.** $f(x) = x(x - 4)^3$

$f'(x) = x\left[3(x - 4)^2\right] + (x - 4)^3 = (x - 4)^2(4x - 4)$

$f''(x) = 4(x - 1)\left[2(x - 4)\right] + 4(x - 4)^2 = 4(x - 4)\left[2(x - 1) + (x - 4)\right] = 4(x$

$f''(x) = 12(x - 4)(x - 2) = 0$ when $x = 2, 4$.

| Test interval: | $-\infty < x < 2$ | $2 < x < 4$ | $4 < x < \infty$ |
|---|---|---|---|
| Sign of $f''(x)$: | $f''(x) > 0$ | $f''(x) < 0$ | $f''(x) > 0$ |
| Conclusion: | Concave upward | Concave downward | Concave upward |

Points of inflection: $(2, -16), (4, 0)$

**27.** $f(x) = x\sqrt{x+3}$

$$f'(x) = x\left(\frac{1}{2}\right)(x+3)^{-1/2} + \sqrt{x+3} = \frac{3(x+2)}{2\sqrt{x+3}}$$

$$f''(x) = \frac{6\sqrt{x+3} - 3(x+2)(x+3)^{-1/2}}{4(x+3)}$$

$$= \frac{3(x+4)}{4(x+3)^{3/2}}$$

$f''(x) > 0$ on the entire domain of $f$ (except for $x = -3$, for which $f''(x)$ is undefined). There are no points of inflection.

Concave upward: $(-3, \infty)$

**31.** $f(x) = \sin\frac{x}{2}, \quad [0, 4\pi]$

$$f'(x) = \frac{1}{2}\cos\left(\frac{x}{2}\right)$$

$$f''(x) = -\frac{1}{4}\sin\left(\frac{x}{2}\right)$$

$f''(x) = 0$ when $x = 0, 2\pi, 4\pi$.

Point of inflection: $(2\pi, 0)$

| Test interval: | $0 < x < 2\pi$ | $2\pi < x < 4\pi$ |
|---|---|---|
| Sign of $f''(x)$: | $f'' < 0$ | $f'' > 0$ |
| Conclusion: | Concave downward | Concave upward |

**35.** $f(x) = 2\sin x + \sin 2x, \quad [0, 2\pi]$

$f'(x) = 2\cos x + 2\cos 2x$

$f''(x) = -2\sin x - 4\sin 2x = -2\sin x(1 + 4\cos x)$

$f''(x) = 0$ when $x = 0, 1.823, \pi, 4.460$.

| Test interval: | $0 < x < 1.823$ | $1.823 < x < \pi$ | $\pi < x < 4.460$ | $4.460 < x < 2\pi$ |
|---|---|---|---|---|
| Sign of $f''(x)$: | $f'' < 0$ | $f'' > 0$ | $f'' < 0$ | $f'' > 0$ |
| Conclusion: | Concave downward | Concave upward | Concave downward | Concave upward |

Points of inflection: $(1.823, 1.452), (\pi, 0), (4.46, -1.452)$

**65.** $f(2) = f(4) = 0$

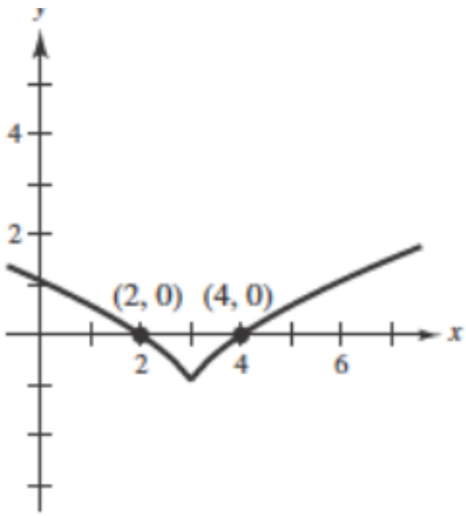
$f'(x) < 0$ if $x < 3$

$f'(3)$ does not exist.

$f'(x) > 0$ if $x > 3$

$f''(x) < 0, x \neq 3$

Possible Solution:




**67.** $f(2) = f(4) = 0$

$f'(x) > 0$ if $x < 3$

$f'(3)$ does not exist.

$f'(x) < 0$ if $x > 3$

$f''(x) > 0, x \neq 3$

Possible Solution: