Calculus Name: Date:

3D: Rolle's Theorem & the Mean Value Theorem

In the last section, we used the *Extreme Value Theorem* to guarantee that a continuous function has a minimum and a maximum value on an interval $[a, b]$. This extreme value could be on the endpoints, or it could be inside the interval. Now we will look at two theorems that tell us more specifically about what must happen inside the interval (a, b).

Rolle's Theorem

Consider This: If we look out our 2nd story building, and a bird flies past the window (going up) then 10 seconds later, it flies past the window on its way down. Did it's height have a maximum during those 10 seconds?

This is the basic idea of a theorem stated by Michel Rolle in 1691.

THEOREM 3.3 ROLLE'S THEOREM

Let f be continuous on the closed interval $[a, b]$ and differentiable on the open interval (a, b). If

$$f(a) = f(b)$$

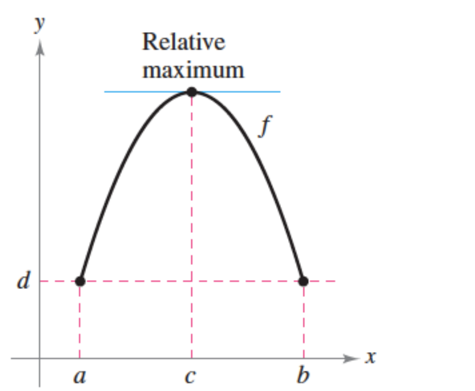
then there is at least one number c in (a, b) such that $f'(c) = 0$.

The key to this simple sounding theorem is the hypotheses:

1. **The function must be continuous,**
2. **The function must be differentiable,**
3. $\boldsymbol{f(a) = f(b)}$

Try It: Use Rolle's theorem to prove that there must be one value c in $(\pi, 2\pi)$ where $f'(c) = 0$ for

$$f(x) = 1 - (x - \pi)^2 \sin x$$



(a) f is continuous on $[a, b]$ and differentiable on (a, b).

Mean Value Theorem

Now we can generalize Rolle's Theorem to get one of the most important theorems in Calculus!

<u>**Consider This**</u>:

6A BOYS

Place	TmPl	No.	Name	Gr	School	Time	Pace
45	37	57	Nathan Burton	12	Grants Pass	16:52	5:26

Nathan was running in the Oregon State Cross-Country Championships, and he ran an average of 5:26 per mile as he and his team blew by about 100 other runners who couldn't keep up!

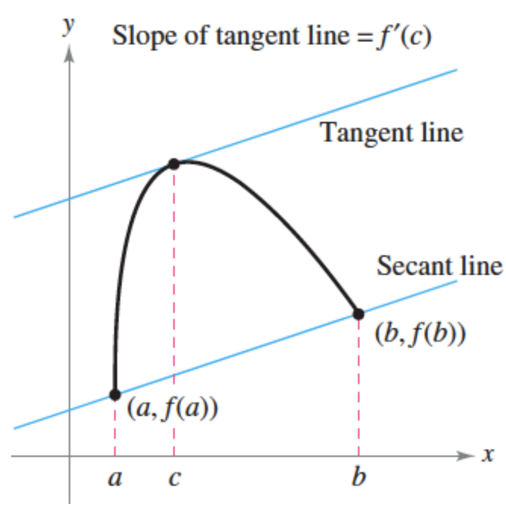
What was his average speed?

Was he ever running slower than this average speed during the race? Was he ever running faster?

Was Nathan's instantaneous speed ever exactly the same as his average speed? How do you know?

This is the main idea of the Mean Value Theorem (a.k.a. MVT). Let's look at it graphically.

Consider the graph to the right.



a) What is the slope of the secant line on (a, b)?
   This is the *average rate of change* on (a, b).

b) Is there a point where the *slope of the tangent line* (i.e. the instantaneous rate of change) is the same as the slope of the secant line?
   Explain how you now in your own words.

THEOREM 3.4 THE MEAN VALUE THEOREM

If f is continuous on the closed interval $[a, b]$ and differentiable on the open interval (a, b), then there exists a number c in (a, b) such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}.$$

Again, the key thing to verify here is the hypotheses:

1. Is $f(x)$ continuous on $[a, b]$?
2. Is $f(x)$ differentiable on (a, b)?

If so, then we can conclude that there is a point on the interval where the

$$\textit{tangent line slope} = \textit{secant line slope}$$

Example 2:

Determine if the MVT applies to $f(x) = x^3 - x$ on $[0,2]$, if so, find the value(s) guaranteed by the theorem.

Example 3:

With the help of your calculator's ability to graphically find zeros, determine all the numbers c which satisfy the conclusion of the Mean Value Theorem for the function $f(x) = x^3 + 2x^2 - x$ on $[-1,2]$.

Example 4:

Determine if the MVT applies to $f(x) = x^3 - 3x^2 + 2x$ on the interval $[0, 3]$. If so, find the values guaranteed by the MVT.