3E Exercises-Solutions

L'Hôpital's Rule

Answer the following questions using the L'Hôpital's Rule

11. $\lim_{x \to 3} \frac{x^2 - 2x - 3}{x - 3}$

$$\lim_{x \to 3} \frac{x^2 - 2x - 3}{x - 3} = \lim_{x \to 3} \frac{2x - 2}{1} = 4$$

13. $\lim_{x \to 0} \frac{\sqrt{25 - x^2} - 5}{x}$

$$\lim_{x \to 0} \frac{\sqrt{25 - x^2} - 5}{x} = \lim_{x \to 0} \frac{\frac{1}{2}(25 - x^2)^{-1/2}(-2x)}{1} = \lim_{x \to 0} \frac{-x}{\sqrt{25 - x^2}} = 0$$

15. $\lim_{x \to 0} \frac{e^x - (1 - x)}{x}$

$$\lim_{x \to 0} \frac{e^x - (1 - x)}{x} = \lim_{x \to 0} \frac{e^x + 1}{1} = 2$$

16. $\lim_{x \to 1} \frac{\ln x^2}{x^2 - 1}$

To be graded on assignment

21. $\lim_{x \to 0} \frac{\sin 3x}{\sin 5x}$

$$\lim_{x \to 0} \frac{\sin 3x}{\sin 5x} = \lim_{x \to 0} \frac{3 \cos 3x}{5 \cos 5x} = \frac{3}{5}$$

23. $\lim_{x \to 0} \frac{\arcsin x}{x}$

$$\lim_{x \to 0} \frac{\arcsin x}{x} = \lim_{x \to 0} \frac{1/\sqrt{1 - x^2}}{1} = 1$$

30. $\lim_{x\to\infty} \frac{x^3}{e^{x^2}}$

To be graded on assignment

51. $\lim_{x\to\infty} x^{1/x}$ (hint: set equal to y and take log)

(a) $\lim_{x\to\infty} x^{1/x} = \infty^0$

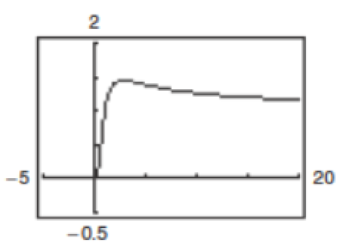
(b) Let $y = \lim_{x\to\infty} x^{1/x}$.

$$\ln y = \lim_{x\to\infty} \frac{\ln x}{x} = \lim_{x\to\infty} \left(\frac{1/x}{1}\right) = 0$$

So, $\ln y = 0 \Rightarrow y = e^0 = 1$. Therefore,

$$\lim_{x\to\infty} x^{1/x} = 1.$$

(c)



59. $\lim_{x\to 2^+}\left(\frac{8}{x^2-4}-\frac{x}{x-2}\right)$ (hint: Simplify expression)

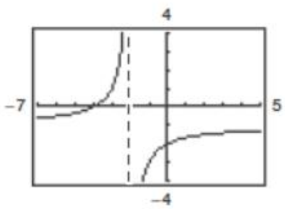
(a) $\lim_{x\to 2^+}\left(\frac{8}{x^2-4}-\frac{x}{x-2}\right) = \infty - \infty$

(b) $\lim_{x\to 2^+}\left(\frac{8}{x^2-4}-\frac{x}{x-2}\right) = \lim_{x\to 2^+}\frac{8-x(x+2)}{x^2-4}$

$= \lim_{x\to 2^+}\frac{(2-x)(4+x)}{(x+2)(x-2)}$

$= \lim_{x\to 2^+}\frac{-(x+4)}{x+2} = \frac{-3}{2}$

(c)



CAPSTONE

88. Determine which of the following limits can be evaluated using L'Hôpital's Rule. Explain your reasoning. Do not evaluate the limit.

(a) $\lim_{x\to 2}\frac{x-2}{x^3-x-6}$

(b) $\lim_{x\to 0}\frac{x^2-4x}{2x-1}$

(c) $\lim_{x\to\infty}\frac{x^3}{e^x}$

(d) $\lim_{x\to 3}\frac{e^{x^2}-e^9}{x-3}$

(e) $\lim_{x\to 1}\frac{\cos \pi x}{\ln x}$

(f) $\lim_{x\to 1}\frac{1+x(\ln x-1)}{\ln x(x-1)}$

a) Yes, approaches $\frac{0}{0}$

b) No, approaches $\frac{0}{1}$

c) Yes, approaches $\frac{\infty}{\infty}$

d) Yes, approaches $\frac{0}{0}$

e) No, approaches $\frac{1}{0}$

f) Yes, approaches $\frac{0}{0}$