

1. (NC) A particle moves according to  $s = t^4 - 4t^3$ , for  $t \geq 0$ , where  $s$  is in meters and  $t$  is in seconds.

a. Find the average velocity during the first second.

$$\frac{\Delta s}{\Delta t} = \frac{s(1) - s(0)}{1 - 0} = \frac{(1-4) - (0-0)}{1} = \frac{-3}{1} = \boxed{-3}$$

Moving left @ 3 m/sec

b. Find the instantaneous velocity at time  $t = 1$ s.

$$V(t) = s'(t) = 4t^3 - 12t^2$$

$$V(1) = 4(1)^3 - 12(1)^2 = \boxed{-8 \text{ m/sec}}$$

c. What is the particle's speed at time  $t = 2$ s?

$$\text{Speed} = |V(2)| = |4(2)^3 - 12(2)^2| = |32 - 48| = |-16| = \boxed{16 \text{ m/sec}}$$

d. Does the particle ever come to a stop? If so, when?

$$V(t) = 0 = 4t^3 - 12t^2$$

$$0 = 4t^2(t - 3)$$

At  $t=0$  and  $t=3$ ,  
the particle stopped

e. When is the particle moving the fastest?

Find  $V(t) = \text{max}$

$$a(t) = V'(t) = 12t^2 - 24t = 0$$

$$12t(t - 2) = 0$$

$$t = 0, 2 \rightarrow \text{local extrema}$$

Since  $V(t) = 4t^3 - 12t^2$

as  $t \rightarrow \infty$ ,  
 $V(t) \rightarrow \infty$   
so no global  
Max  $V$  exists

f. Find the distance traveled by the particle during the first 5 seconds.

Since the particle stopped at 3 seconds, we must use the intervals  $[0, 3]$  and  $[3, 5]$

on  $[0, 3]$   $d_1 = |s(3) - s(0)| = |-27 - 0| = 27$

$d_2 = |s(5) - s(3)| = |125 - (-27)| = 152$

Total =  $27 + 152 = \boxed{179 \text{ m}}$

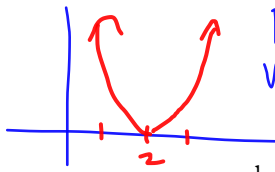
$$s(0) = 0$$

$$s(3) = (3)^4 - 4(3)^3 = 81 - 108 = -27$$

$$s(5) = (5)^4 - 4(5)^3 = 625 - 500 = 125$$

2. (NC) The position of an object moving along a straight line is given by  $s = t^3 - 6t^2 + 12t - 8$ , for  $t \geq 0$  where  $s$  is in feet and  $t$  is in seconds.

- a. Find the interval during which the <sup>position</sup> displacement is increasing.



Find  $V(t) > 0$   
 $V(t) = S'(t) = 3t^2 - 12t + 12 = 0$   
 $3(t^2 - 4t + 4) = 0$   
 $(t-2)^2 = 0 \quad t=2$

Velocity is positive  
 on  $(-\infty, 2) \cup (2, \infty)$

- b. Find the interval during which the velocity is decreasing.

On  $(-\infty, 2)$

- c. What is the minimum value of the speed of the particle?

at  $t=2$ ,  $|V|=0$ . This gives  
 us a minimum speed of 0 mph

3. (C) A particle moves along a line according to  $s = 2t^3 - 9t^2 + 12t - 4$ ,  $t \geq 0$  where  $s$  is in meters and  $t$  is in seconds.

- a. At what values of  $t$  is the <sup>position</sup> displacement increasing?

$V(t) = S'(t) = 6t^2 - 18t + 12 = 0$   
 $6(t^2 - 3t + 2) = 0$   
 $6(t-2)(t-1) = 0$

at  $t=2$  and  $t=1$   
 $V(t) = 0$



So,  $V(t) > 0$  and position is increasing on  $(0, 1)$  and  $(2, \infty)$

- b. At what values of  $t$  is the velocity increasing?

Vertex of  $V(t)$  at  $t = \frac{3}{2}$

So,  $V(t)$  is increasing on  $(\frac{3}{2}, \infty)$

- c. What is the particle's speed when  $t=1.5$ s?

$|V(1.5)| = |6(\frac{3}{2})^2 - 18(\frac{3}{2}) + 12| = |-15| = 15 \text{ m/sec}$

- d. What is the total distance traveled between  $t=0$  and  $t=4$ ?

Since  $V(t) = 0$  at  $t=1$  &  $t=2$ ,  
 find distance traveled on  
 intervals  $(0, 1)$   $(1, 2)$   $(2, 4)$

$S(0) = -4$

$S(1) = 1$

$S(2) = 0$

$S(4) = 28$

$d_1 = |S(1) - S(0)| = |1 - (-4)| = 5$

$d_2 = |S(2) - S(1)| = |0 - 1| = |-1| = 1$

$d_3 = |S(4) - S(2)| = |28 - 0| = 28$

Total distance =  $5 + 1 + 28 = 34 \text{ m}$