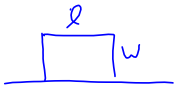
Calculus Name: Date:

Unit 3 Test 2 Review

Optimization

1. A farmer wants to fence in a rectangular pen so that is against a building. If he has 100 feet of fencing for the 3 sides of the pen, find the dimensions of the pen that will give him a *maximum area* in the pen.



$A = lw$

$100 = 2w + l$

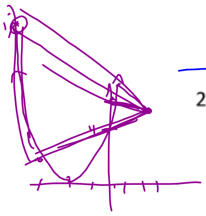
$l = 100 - 2w$

$A = (100-2w)(w)$

$A = 100w - 2w^2$

$A' = 100 - 4w = 0$

$\boxed{w = 25\text{ ft} \quad l = 50\text{ ft}}$



2. Find the point on the graph of $y = (x+2)^2$ that is closest to the point (3,5).

$d = \sqrt{(x-3)^2 + (y-5)^2}$

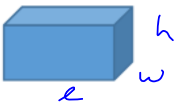
$d = \sqrt{(x-3)^2 + ((x+2)^2 - 5)^2}$

$d'(x) = 0$

$x = -3.7, -2.6, \boxed{.358}$

$\boxed{(.358, 5.560}$

3. A rectangular prism has a base that has a length that is twice the width. If the surface area of the box is 200 in^2, find the dimensions of the box that will *maximize* the volume.



$V = lwh = (2w)w\left(\frac{200-4w^2}{6w}\right)$

$A = 2lw + 2wh + 2lh \quad l = 2w$

$A = 2(2w)w + 2wh + 2(2w)h$

$A = 4w^2 + 6wh$

$200 = 4w^2 + 6wh \rightarrow h = \frac{200-4w^2}{6w}$

$V = \frac{w(200-4w^2)}{3}$

$V = \frac{200}{3}w - \frac{4}{3}w^3$

$V' = \frac{200}{3} - 4w^2 = 0$

$\sqrt{w^2} = \sqrt{\frac{50}{3}}$

$\boxed{w = \sqrt{50/3}}$

$w \approx 4.082$

$= \frac{5\sqrt{6}}{3}$

$l = 2 \cdot \frac{5\sqrt{6}}{3} = \frac{10\sqrt{6}}{3} \approx 8.164$

$h = \frac{200 - 4(\sqrt{50/3})^2}{6(\sqrt{50/3})} = \frac{200 - \frac{200}{3}}{30\sqrt{6}/3} = \frac{600-200}{30\sqrt{6}} = \frac{40}{3\sqrt{6}} = 5.443$

** $\boxed{\text{Solution: } 8.164 \times 4.082 \times 5.433}$

One Dimensional Movement

4\.

Two particles move along the x-axis. For $0 \le t \le 8$, the position of particle P at time t is given by $x_P(t) = \ln(t^2 - 2t + 10)$, while the velocity of particle Q at time t is given by $v_Q(t) = t^2 - 8t + 15$. Particle Q is at position $x = 5$ at time $t = 0$.

(a) For $0 \le t \le 8$, when is particle P moving to the left?

$x_p'(t) = V_p(t) = \frac{2t-2}{t^2-2t+10} = 0 \longrightarrow 2t - 2 = 0$

$t = 1$

$V_p \xleftarrow{\quad - \quad} \underset{1}{|} \xrightarrow{\quad + \quad}$

$\boxed{\text{Moving Left when } 0 < t < 1}$

(b) For $0 \le t \le 8$, find all times t during which the two particles travel in the same direction.

$V_Q(t) = t^2 - 8t + 15$

$= (t-5)(t-3) = 0 \longrightarrow t = 5, 3$

$V_Q \xleftarrow{\quad + \quad} \underset{3}{|} \quad - \quad \underset{5}{|} \xrightarrow{\quad + \quad}$

$\boxed{\text{Same direction when } t > 5}$

(c) Find the acceleration of particle Q at time $t = 2$. Is the speed of particle Q increasing, decreasing, or neither at time $t = 2$? Explain your reasoning.

$a_Q(t) = V_Q'(t) = 2t - 8$

$a_Q(2) = 2(2) - 8 = -4$ ← This means it is being accelerated toward Left

$V_Q(2) = (2)^2 - 8(2) + 15 = 3$ ← This means it is moving Right

$\boxed{\text{So, the speed is decreasing because acceleration \& Velocity are opposite directions}}$

5.

For $t \geq 0$, a particle moves along the x-axis. The velocity of the particle at time t is given by $v(t) = 1 + 2\sin\left(\frac{t^2}{2}\right)$. The particle is at position $x = 2$ at time $t = 4$.

(a) At time $t = 4$, is the particle speeding up or slowing down?

$a(t) = v'(t) = 1 + 2(t)\cos\left(\frac{t^2}{2}\right)$

$a(4) = 1 + 2(4)\cos\left(\frac{4^2}{2}\right)$

$= 1 + 8\cos(8) = -.164$ (Pulling Left)

$V(4) = 1 + 2\sin\left(\frac{4^2}{2}\right) \approx 2.97$ (Right)

$\boxed{\text{It's Slowing down}}$

(b) Find all times t in the interval $0 < t < 3$ when the particle changes direction. Justify your answer.

✓

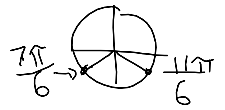
$V(t) = 1 + 2\sin\left(\frac{t^2}{2}\right) = 0$

$\sin\left(\frac{t^2}{2}\right) = \frac{-1}{2}$

$\frac{t^2}{2} = \frac{7\pi}{6} \rightarrow t = \sqrt{\frac{7\pi}{3}} = 2.707$

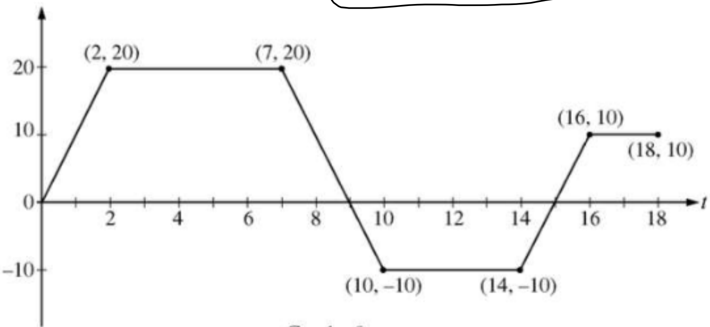
or

$\frac{t^2}{2} = \frac{11\pi}{6} \rightarrow t = \sqrt{\frac{11\pi}{3}} = 3.\cancel{394}$



$\boxed{t = \sqrt{\frac{7\pi}{3}} \approx 2.707 \text{ sec.}}$

6.



Graph of v

A squirrel starts at building A at time $t = 0$ and travels along a straight, horizontal wire connected to building B. For $0 \leq t \leq 18$, the squirrel's velocity is modeled by the piecewise-linear function defined by the graph above.

(a) At what times in the interval $0 < t < 18$, if any, does the squirrel change direction? Give a reason for your answer.

$t = 9$ and $t = 15$

because Velocity changes sign (+/−)

(b) At what time in the interval $0 \leq t \leq 18$ is the squirrel farthest from building A?

at $t = 9$ because

he is moving forward for

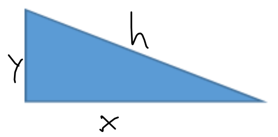
first 2 seconds

then still moving forward for 2

Seconds from 7s to 9s.

Related Rates

7. The longer leg of a right triangle is shrinking at 2 inches per second, and the shorter leg of the triangle is stretching at 1 inch per second. Find the rate of change of the hypotenuse when the longer leg is 10 inches and the shorter leg is 7 inches.



$$\frac{dx}{dt} = -2$$

$$\frac{dy}{dt} = 1$$

$$h = \sqrt{x^2+y^2}$$

$$\frac{dh}{dt} = \frac{2x\frac{dx}{dt}+2y\frac{dy}{dt}}{2\sqrt{x^2+y^2}} = \frac{2(10)(-2)+2(7)(1)}{2\sqrt{10^2+7^2}}$$

$$= \frac{-26}{2\sqrt{149}} = \frac{-13}{\sqrt{149}} \approx \boxed{-1.065 \text{ in/sec}}$$

8. The radius of r of a sphere is increasing at a rate of 3 inches per second. Find the change in the volume $V = \frac{4}{3}\pi r^3$ when the radius is $\boxed{\text{6 inches.}}$

$$\frac{dr}{dt} = 3$$

$$\frac{dv}{dt} = 4\pi r^2 \frac{dr}{dt}$$

$$= 4\pi(6)^2(3)$$

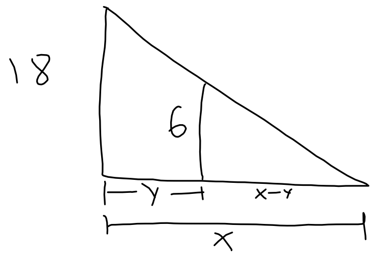
36 12 36 12

$$= \boxed{432\pi \text{ in}^3/\text{sec}}$$

9. A man 6 feet tall walks at a rate of 3 feet per second away from a light that is 18 feet above the ground. When he is 12 feet from the base of the light, at what rate is the tip of the shadow moving?

36

$$\frac{dy}{dt} = 3 \text{ ft/sec}$$



$$\frac{18}{x} = \frac{6}{x-y}.$$

$$18x - 18y = 6x$$

$$18x - 6x = 18y$$

$$12x = 18y$$

$$x = \frac{18}{12}y \longrightarrow x = \frac{3}{2}y$$

$$\frac{dx}{dt} = \frac{3}{2}\frac{dy}{dt}$$

$$\frac{dx}{dt} = \frac{3}{2}\cdot(3) = \frac{9}{2} = \boxed{4.5 \text{ ft/sec}}$$