Calculus

Name: Date:

4A.1: Approximating Area - Practice

Actual Areas $\boxed{f(x) \to 16}$ $\boxed{g(x) = \frac{26}{3}} = 8\frac{2}{3}$

Approximating Area

1. Approximate the area under the curve $f(x) = \frac{1}{4}x(x-1)(x-2)+3$ on the interval [0,4] using the Left-Hand, Right-Hand, Midpoint, and Trapezoidal Sums with 4 sub-intervals. Draw your rectangles or trapezoids on the graphs and find an exact approximations (is that an oxymoron?)

   a. Left-Hand Sum

$f(x) = \frac{1}{4}x^3 - \frac{3}{4}x^2 - \frac{1}{2}x + 3$

$f(0) = 3$ $f(1) = 3$ $f(2) = 3$ $f(3) = \frac{6}{4}+3 = 4\frac{1}{2}$ $f(4) = \frac{1}{4} \cdot 4 \cdot 3 \cdot 2 + 3 = 9$

$LHS = f(0) \cdot 1 + f(1) \cdot 1 + f(2) \cdot 1 + f(3) \cdot 1$

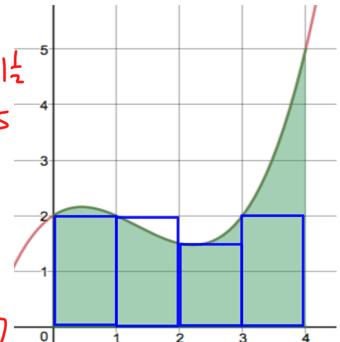
$= 3+3+3+4\frac{1}{2} = \boxed{13.5}$

$g(x) = \frac{1}{4}x(x-1)(x-3)+2$

$g(0) = 2$ $g(1) = 2$ $g(2) = \frac{1}{4} \cdot 2 \cdot 1 \cdot (-1) + 2 = 1\frac{1}{2}$

$g(3) = 2$ $g(4) = \frac{1}{4} \cdot 4(3)(1) + 2 = 5$

$LHS = 2 + 2 + 1\frac{1}{2} + 2 = \boxed{7.5}$



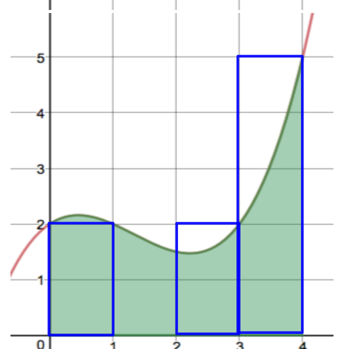
   b. Right-Hand Sum

$RHS = f(1) + f(2) + f(3) + f(4)$

$= 3 + 3 + 4\frac{1}{2} + 9$

$= \boxed{19.5}$

$RHS = 2 + 1\frac{1}{2} + 2 + 5 = \boxed{10.5}$



   c. Midpoint Sum

$f(\frac{1}{2}) = \frac{1}{4}(\frac{1}{2})(\frac{1}{2}-1)(\frac{1}{2}-2)+3 = \frac{-1}{16} \cdot \frac{-3}{2} + 3 = 3\frac{3}{32}$

$f(\frac{3}{2}) = \frac{1}{4}(\frac{3}{2})(\frac{3}{2}-1)(\frac{3}{2}-2)+3 = \frac{3}{8}(\frac{1}{2})(\frac{-1}{2})+3 = 2\frac{29}{32}$

$f(\frac{5}{2}) = \frac{1}{4}(\frac{5}{2})(\frac{5}{2}-1)(\frac{5}{2}-2)+3 = \frac{5}{8}(\frac{3}{2})(\frac{1}{2})+3 = 3\frac{15}{32}$

$f(\frac{7}{2}) = \frac{1}{4}(\frac{7}{2})(\frac{7}{2}-1)(\frac{7}{2}-2)+3 = \frac{7}{8}(\frac{5}{2})(\frac{3}{2})+3 = 3\frac{105}{32} = 6\frac{9}{32}$

$MPS = 14\frac{56}{32} = \boxed{15\frac{24}{32}}$

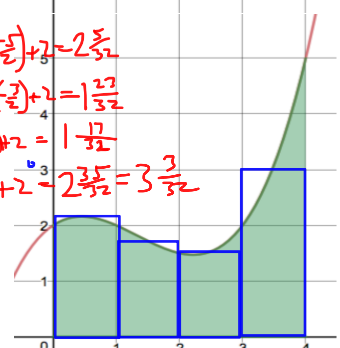
$f(\frac{1}{2}) = \frac{1}{4}(\frac{1}{2})(\frac{1}{2}-1)(\frac{1}{2}-3)+2 = \frac{1}{8} \cdot (\frac{-1}{2})(\frac{-5}{2})+2 = 2\frac{5}{32}$

$f(\frac{3}{2}) = \frac{1}{4}(\frac{3}{2})(\frac{3}{2}-1)(\frac{3}{2}-3)+2 = \frac{3}{8}(\frac{1}{2})(\frac{-3}{2})+2 = 1\frac{23}{32}$

$f(\frac{5}{2}) = \frac{1}{4}(\frac{5}{2})(\frac{5}{2}-1)(\frac{5}{2}-3)+2 = \frac{5}{8}(\frac{3}{2})(\frac{-1}{2})+2 = 1\frac{17}{32}$

$f(\frac{7}{2}) = \frac{1}{4}(\frac{7}{2})(\frac{7}{2}-1)(\frac{7}{2}-3)+2 = \frac{7}{8}(\frac{5}{2})(\frac{1}{2})+2 = 2\frac{35}{32} = 3\frac{3}{32}$

$MPS = 7\frac{48}{32} = \boxed{8\frac{16}{32}}$



   d. Trapezoidal Sum

$TS = \frac{1}{2}\left((f(0)+f(1)) + (f(1)+f(2)) + (f(2)+f(3)) + (f(3)+f(4))\right)$

$= \frac{1}{2}(3 + 3 + 3 + 3 + 3 + 4\frac{1}{2} + 4\frac{1}{2} + 9)$

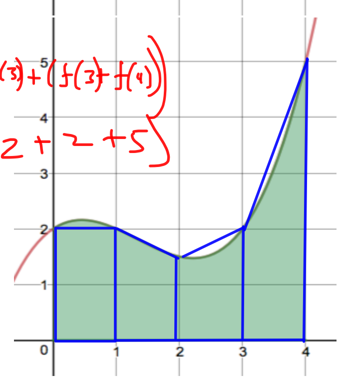
$= \frac{1}{2}(33) = \boxed{16\frac{1}{2}}$

$TS = \frac{1}{2}\left((f(0)+f(1)) + (f(1)+f(2)) + (f(2)+f(3)) + (f(3)+f(4))\right)$

$TS = \frac{1}{2}(2 + 2 + 2 + 1\frac{1}{2} + 1\frac{1}{2} + 2 + 2 + 5)$

$TS = \frac{1}{2}(18)$

$TS = \boxed{9}$



Exact Area $5 - \cos(4) \approx 5.654$

2. Approximate the area under the curve $f(x) = \sin(x) + 1$ on the interval $[0, \pi]$ using the Left-Hand, Right-Hand, Midpoint, and Trapezoidal Sums with 4 sub-intervals. Draw your rectangles or trapezoids on the graphs and find an exact and decimal approximations.
Note: $A = \frac{\pi}{4} \cdot h_1 + \frac{\pi}{4} \cdot h_2 + \frac{\pi}{4} \cdot h_3 + \frac{\pi}{4} \cdot h_4 = \frac{\pi}{4}(h_1 + h_2 + h_3 + h_4)$ for rectangle heights h_i.

a. Left-Hand Sum

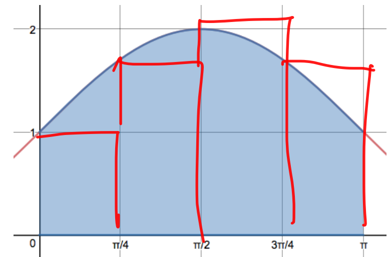
$f(0)=1 \quad f(\frac{\pi}{4}) = \frac{\sqrt{2}}{2}+1 \quad f(\frac{\pi}{2}) = 1+1=2$

$f(\frac{3\pi}{4}) = \frac{\sqrt{2}}{2}+1 \quad f(\pi) = 0+1=1$

$LHS = \frac{\pi}{4}\left(1 + \frac{\sqrt{2}}{2} + 1 + 2 + \frac{\sqrt{2}}{2} + 1\right)$

$= \frac{\pi}{4}(5+\sqrt{2})$

≈ 5.038



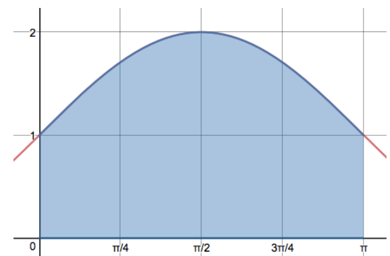
$f(x) = \sin x + 1$

b. Right-Hand Sum

$RHS = \frac{\pi}{4}\left(\frac{\sqrt{2}}{2} + 1 + 2 + \frac{\sqrt{2}}{2} + 1 + 1\right)$

$= \frac{\pi}{4}(5+\sqrt{2})$

≈ 5.038



$f(x) = \sin x + 1$

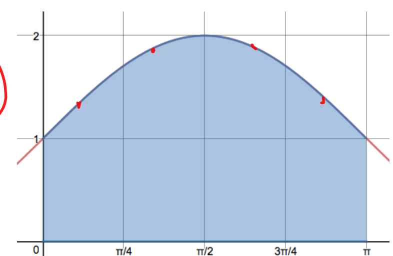
c. Midpoint Sum (decimal only) $\quad \sin\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1-\cos\theta}{2}}$

$MPS = \frac{\pi}{4}\left(f\left(\frac{\pi}{8}\right) + \underbrace{f\left(\frac{3\pi}{8}\right) + f\left(\frac{5\pi}{8}\right)}_{\text{Same!}} + f\left(\frac{7\pi}{8}\right)\right)$

Same!

$= \frac{\pi}{4}\left(2\sin\frac{\pi}{8} + 2\sin\left(\frac{3\pi}{8}\right) + 4\right)$

≈ 5.19



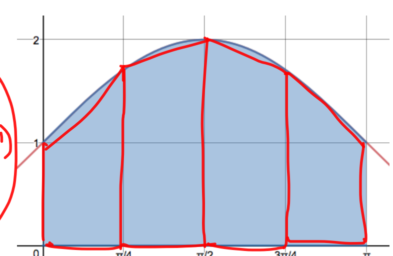
$f(x) = \sin x + 1$

d. Trapezoidal Sum (decimal only)

$TS = \frac{1}{2} \cdot \frac{\pi}{4}\left(\left(1+\frac{\sqrt{2}}{2}+1\right) + \left(\frac{\sqrt{2}}{2}+1+2\right) + \left(2+\frac{\sqrt{2}}{2}+1\right) + \left(\frac{\sqrt{2}}{2}+1+1\right)\right)$

$= \frac{1}{2} \cdot \frac{\pi}{4}\left(2 \cdot \left(\left(2+\frac{\sqrt{2}}{2}\right) + \left(3+\frac{\sqrt{2}}{2}\right)\right)\right)$

$= \boxed{\frac{\pi}{4}(5+\sqrt{2})} \approx \boxed{5.038}$



$f(x) = \sin x + 1$