4C: Antiderivatives and Indefinite Integration

Now that we have defined a definite integral using limits and summation, you're probably wishing there was an easier way to find an integral, and there is… it's called the anti-derivative. We will see the connection between the summations and the antiderivative when we discuss the Fundamental Theorem of Calculus later.

Exploration

For each derivative, describe the original function F.

a. $F'(x) = 2x$

b. $F'(x) = x$

c. $F'(x) = x^2$

d. $F'(x) = \frac{1}{x^2}$

e. $F'(x) = \frac{1}{x^3}$

f. $F'(x) = \cos x$

Antiderivatives

Definition of Antiderivative

A function F is an **antiderivative** of f on an interval I if $F'(x) = f(x)$ for all x in I

Example: What is the antiderivative of $f(x) = 3x^2$?

Now, notice that all these functions all have the property that $F'(x) = 3x^2$

$$F(x) = x^3, \qquad F(x) = x^3 + 1, \qquad F(x) = x^3 + \pi, \qquad F(x) = x^3 - 237.5e^{43}$$

So, it is possible that the real function has some constant attached to it. We call this the **Constant of Integration.** So, we have to call this unknown value C and we say that

$$\text{the antiderivative of } f(x) = 3x^2 \text{ is } F(x) = x^3 + C$$

So, if you know your derivatives well, you can find Antiderivatives by thinking backwards.

Indefinite Integral

An equation with a differential in it is called a **differential equation** that compares dy and dx.

So, we can come up with some notation by solving this differential equation:

$$\frac{dy}{dx} = f(x) \quad \rightarrow \quad dy = f(x)\,dx$$

To solve for y, we need to do the opposite of differentiation, so we need to find the *andtiderivative.*

$$y = \int f(x)\,dx \; = F(x) + C$$

This is what we call an ***indefinite integral.*** Now we see that integration (the process of finding the indefinite integral) is the *inverse* of differentiation. So, we can say

$$\frac{d}{dx}\left(\int f(x)\,dx\right) = f(x)$$

Let's find some more integrals

Examples Find these antiderivatives.

a. $\displaystyle\int x^3\,dx$

b. $\displaystyle\int 5\,dx$

c. $\displaystyle\int x^3 + x^2\,dx$

BASIC INTEGRATION RULES

Differentiation Formula	*Integration Formula*	
$\frac{d}{dx}[C] = 0$	$\int 0\,dx = C$	
$\frac{d}{dx}[kx] = k$	$\int k\,dx = kx + C$	
$\frac{d}{dx}[kf(x)] = kf'(x)$	$\int kf(x)\,dx = k\int f(x)\,dx$	
$\frac{d}{dx}[f(x) \pm g(x)] = f'(x) \pm g'(x)$	$\int [f(x) \pm g(x)]\,dx = \int f(x)\,dx \pm \int g(x)\,dx$	
$\frac{d}{dx}[x^n] = nx^{n-1}$	$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1$	Power Rule
$\frac{d}{dx}[\sin x] = \cos x$	$\int \cos x\,dx = \sin x + C$	
$\frac{d}{dx}[\cos x] = -\sin x$	$\int \sin x\,dx = -\cos x + C$	
$\frac{d}{dx}[\tan x] = \sec^2 x$	$\int \sec^2 x\,dx = \tan x + C$	
$\frac{d}{dx}[\sec x] = \sec x \tan x$	$\int \sec x \tan x\,dx = \sec x + C$	
$\frac{d}{dx}[\cot x] = -\csc^2 x$	$\int \csc^2 x\,dx = -\cot x + C$	
$\frac{d}{dx}[\csc x] = -\csc x \cot x$	$\int \csc x \cot x\,dx = -\csc x + C$	

__Example__ Use the properties of antiderivatives to find the integrals.

$$\int 3x^2 \; dx$$

$$\int \frac{1}{x^3} \; dx$$

$$\int \sqrt{x} \; dx$$

$$\int 3x^4 - 5x^2 + x \, dx$$