Calculus Name: Date:

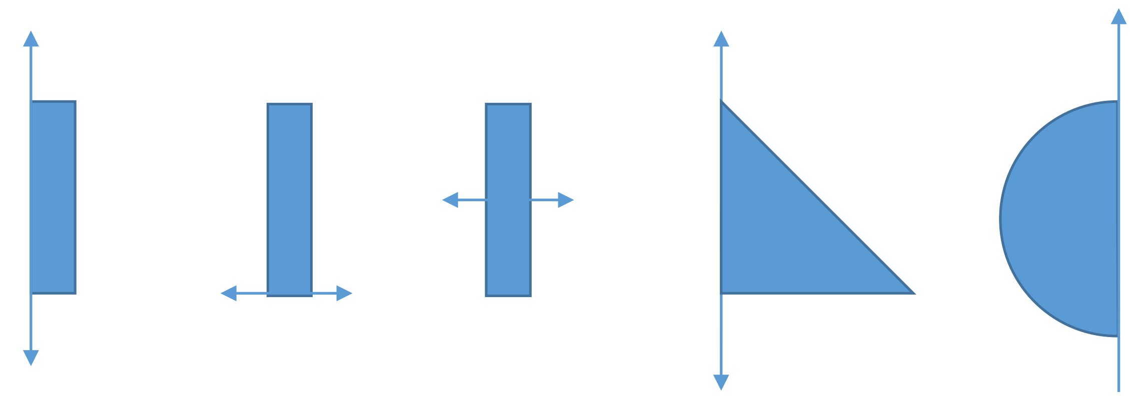
5D Volume

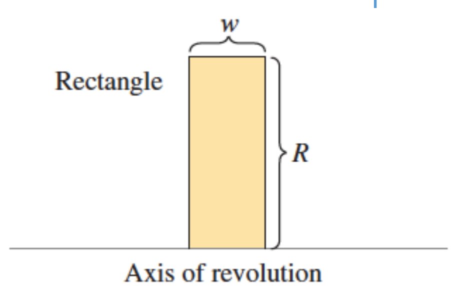
In the previous section we saw how it is useful to find to subtracting functions can give us the area between to curves. We will now take this a dimension higher to use integration to find volume of a solid.

Disk Method

When a 2 dimensional shape is spun around an **axis of revolution**, we get a **solid of revolution.**

Draw a picture of the 3D solids do you get if you spin the 2D shapes around the given axis of revolution.

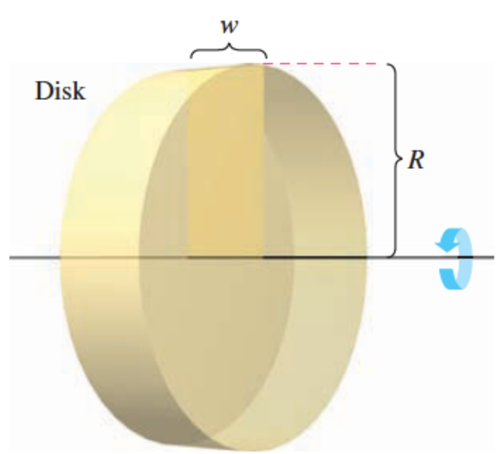




The shapes that we will focus on are the rectangles because an integral is just a sum of infinitely thin rectangles.

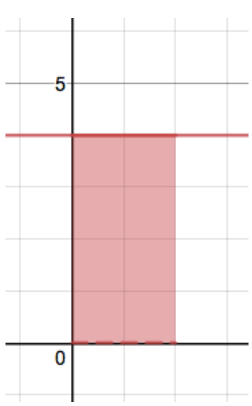
When finding the volume of a disk (i.e. a thin cylinder) we will use the formula

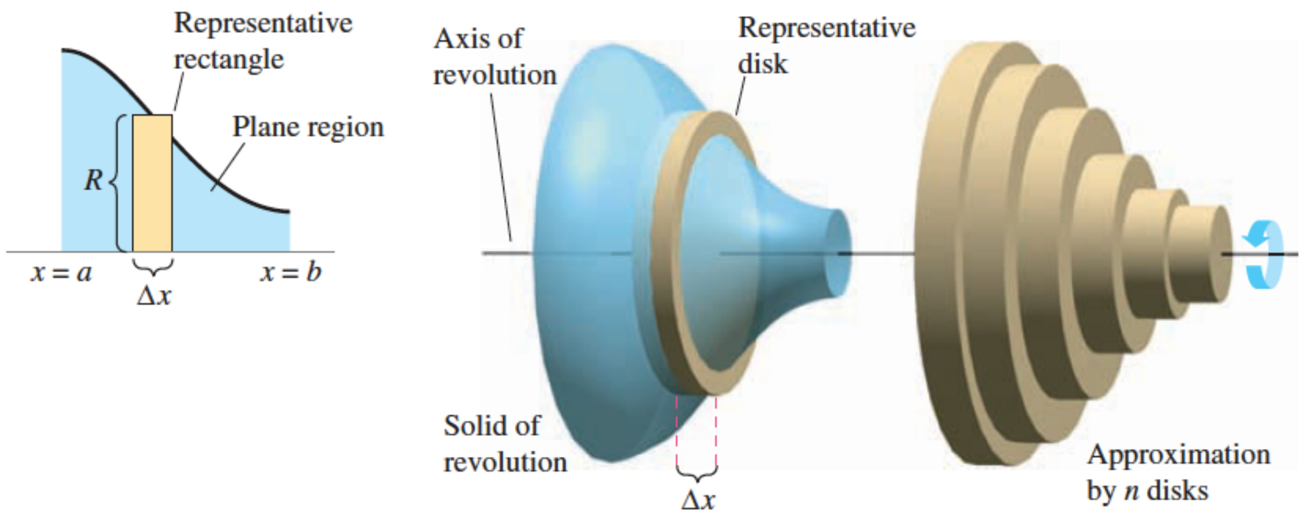
$$Volume = (Area\ of\ disk)(width\ of\ the\ disk) = \pi R^2 w$$



Volume of a disk: $\pi R^2 w$

__Consider This:__ Find the volume of a function $R(x) = 4$ that we want to revolve around the x-axis.





With the Disk Method we want to find the volume of many disks with radius $R(x)$ and the width Δx, and we find the limit as $\Delta x \to 0$.

This gives us

$$Volume\ of\ Solid = \lim_{\Delta x \to 0} \pi \sum_{i=1}^{n} [R(x_i)]^2 \Delta x = \pi \int_a^b [R(x)]^2 dx$$

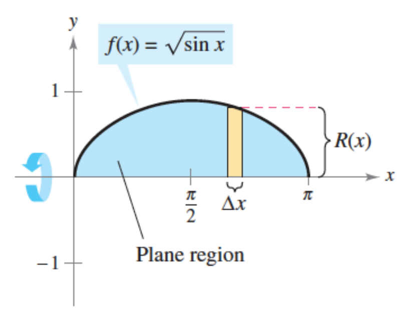
THE DISK METHOD

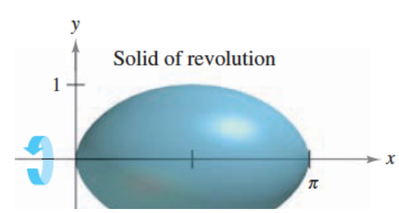
To find the volume of a solid of revolution with the **disk method,** use one of the following, as shown in Figure 7.15.

Horizontal Axis of Revolution	*Vertical Axis of Revolution*
Volume $= V = \pi \int_a^b [R(x)]^2\, dx$	Volume $= V = \pi \int_c^d [R(y)]^2\, dy$

Example

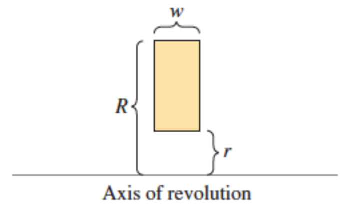
Find the volume of solid formed by the region bounded by the graph of $f(x) = \sqrt{\sin x}$ and the x-axis $(0 \le x \le \pi)$ about the x-axis.



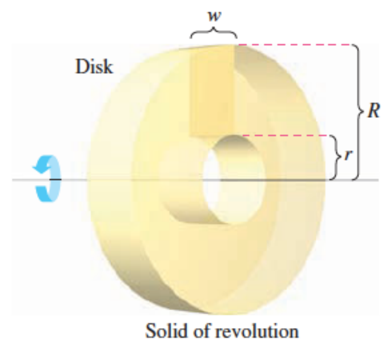


The Washer Method

If the region to be revolved is not adjacent to the axis of revolution, then we need to use the ***Washer Method*** to find the volume.



The key to the washer method is viewing the volume as a large cylinder minus a small cylinder.

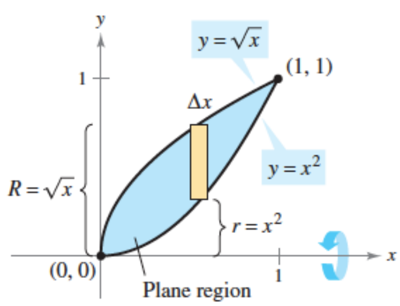


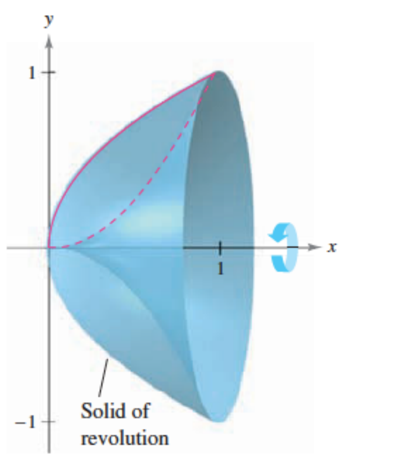
$$V = \pi \int_a^b \left([R(x)]^2 - [r(x)]^2\right) dx.$$

z

__Example__

Find the volume of the solid formed by revolving the region bounded by the graph of $y = \sqrt{x}$ and $y = x^2$ about the x −axis.





Solid of revolution

Solids with Known Cross Sections

With the disk method, you can find the volume of a solid having a circular cross section whose area is $A = \pi R^2$. This method can be generalized to solids of any shape, as long as you know a formula for the area of an arbitrary cross section. Some common cross sections are squares, rectangles, triangles, semicircles, and trapezoids.

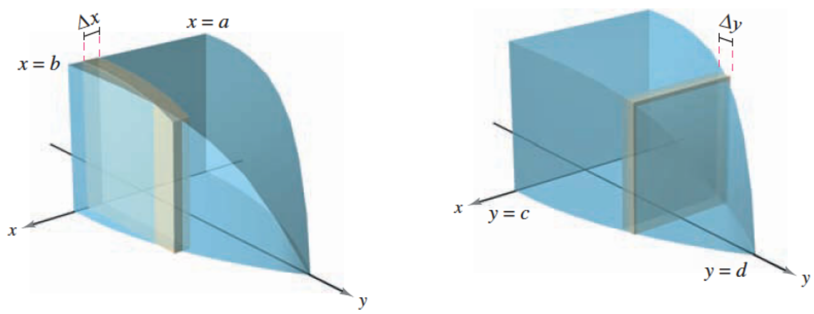
VOLUMES OF SOLIDS WITH KNOWN CROSS SECTIONS

1. For cross sections of area $A(x)$ taken perpendicular to the x-axis,

$$\text{Volume} = \int_a^b A(x)\,dx.$$ See Figure 7.24(a).

2. For cross sections of area $A(y)$ taken perpendicular to the y-axis,

$$\text{Volume} = \int_c^d A(y)\,dy.$$ See Figure 7.24(b).



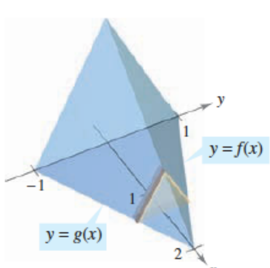
(a) Cross sections perpendicular to x-axis **(b)** Cross sections perpendicular to y-axis

Example

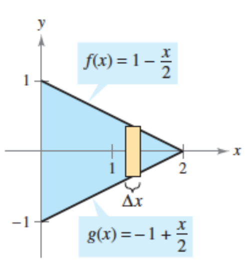
Find the volume of the solid shown in Figure 7.25. The base of the solid is the region bounded by the lines

$$f(x) = 1 - \frac{x}{2}, \qquad g(x) = -1 + \frac{x}{2}, \qquad \text{and} \qquad x = 0.$$

The cross sections perpendicular to the x-axis are equilateral triangles.



Cross sections are equilateral triangles.



Triangular base in xy-plane