Calculus Name: Date:

AP Calculus A/B Practice Test #2

Multiple Choice (Goal: 2-3 min. each)

NC

11. At time t, a population of bacteria grows at the rate of $5e^{0.2t}+4t$ grams per day, where t is measured in days. By how many grams has the population grown from time $t=0$ days to $t=10$ days?

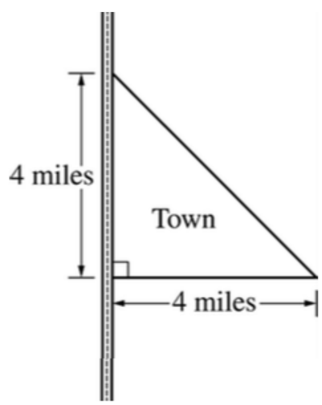
(A) $5e^2+40$

(B) $5e^2+195$

(C) $25e^2+175$

(D) $25e^2+375$

NC



12. The right triangle shown in the figure above represents the boundary of a town that is bordered by a highway. The population density of the town at a distance of x miles from the highway is modeled by $D(x)=\sqrt{x+1}$, where $D(x)$ is measured in thousands of people per square mile. According to the model, which of the following expressions gives the total population, in thousands, of the town?

(A) $\int_0^4 \sqrt{x+1}\,dx$

(B) $\int_0^4 8\sqrt{x+1}\,dx$

(C) $\int_0^4 x\sqrt{x+1}\,dx$

(D) $\int_0^4 (4-x)\sqrt{x+1}\,dx$

NC 13. Which of the following is the solution to the differential equation $\frac{dy}{dx} = y\sec^2 x$ with the initial condition $y\left(\frac{\pi}{4}\right) = -1$?

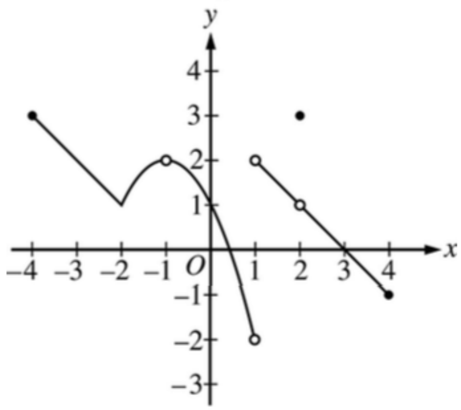
(A) $y = -e^{\tan x}$

(B) $y = -e^{(-1+\tan x)}$

(C) $y = -e^{\left(\sec^3 x - 2\sqrt{2}\right)/3}$

(D) $y = -\sqrt{2\tan x - 1}$

NC



Graph of f

14. The graph of the function f is shown in the figure above. For how many values of x in the open interval $(-4, 4)$ is f discontinuous?

(A) one

(B) two

(C) three

(D) four

NC

15.

x	0	1	2
$f(x)$	5	2	-7
$f'(x)$	-2	-5	-14

The table above gives selected values of a differentiable and decreasing function *f* and its derivative. If *g* is the inverse function of *f*, what is the value of $g'(2)$?

(A) $-\frac{1}{5}$

(B) $-\frac{1}{14}$

(C) $\frac{1}{5}$

(D) 5

CR

16. The derivative of the function *f* is given by $f'(x) = -\frac{x}{3} + \cos\left(x^2\right)$. At what values of *x* does *f* have a relative minimum on the interval $0 < x < 3$?

(A) 1.094 and 2.608

(B) 1.798

(C) 2.372

(D) 2.493

CR

17. The second derivative of a function *g* is given by $g''(x) = 2^{-x^2} + \cos x + x$. For $-5 < x < 5$, on what open intervals is the graph of *g* concave up?

(A) $-5 < x < -1.016$ only

(B) $-1.016 < x < 5$ only

(C) $0.463 < x < 2.100$ only

(D) $-5 < x < 0.463$ and $2.100 < x < 5$

CR

18. The temperature, in degrees Fahrenheit $(°\mathrm{F})$, of water in a pond is modeled by the function *H* given by $H(t) = 55 - 9\cos\left(\frac{2\pi}{365}(t+10)\right)$, where *t* is the number of days since January 1 $(t = 0)$. What is the instantaneous rate of change of the temperature of the water at time $t = 90$ days?

(A) 0.114°F/day

(B) 0.153°F/day

(C) 50.252°F/day

(D) 56.350°F/day

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CR

19.

x	0	2	4	8
$f(x)$	3	4	9	13
$f'(x)$	0	1	1	2

The table above gives values of a differentiable function f and its derivative at selected values of x. If h is the function given by $h(x) = f(2x)$, which of the following statements must be true?

(I) h is increasing on $2 < x < 4$.

(II) There exists c, where $0 < c < 4$, such that $h(c) = 12$.

(III) There exists c, where $0 < c < 2$, such that $h'(c) = 3$.

(A) II only

(B) I and III only

(C) II and III only

(D) I, II, and III

CR

20. Let h be the function defined by $h(x) = \dfrac{1}{\sqrt{x^5 + 1}}$. If g is an antiderivative of h and $g(2) = 3$, what is the value of $g(4)$?

(A) -0.020

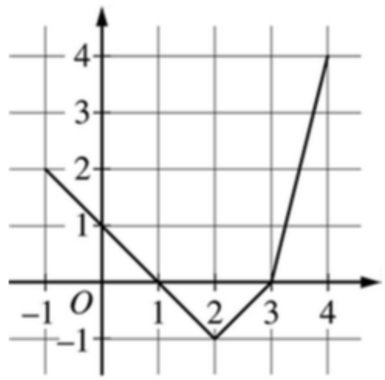
(B) 0.152

(C) 3.031

(D) 3.152

Free Response (Goal: 15 min. each)

CR



Graph of f

3. Let f be a continuous function defined on the closed interval $-1 \le x \le 4$. The graph of f, consisting of three line segments, is shown above. Let g be the function defined by $g(x) = 5 + \int_2^x f(t)\, dt$ for $-1 \le x \le 4$.

   (A) Find $g(4)$.

   (B) On what intervals is g increasing? Justify your answer.

   (C) On the closed interval $-1 \le x \le 4$, find the absolute minimum value of g and find the absolute maximum value of g. Justify your answers.

   (D) Let $h(x) = x \cdot g(x)$. Find $h'(2)$.

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CR 4.

An airplane takes off along a straight runway. For $0 \le t \le 20$, the plane's velocity is given by a differentiable function v. Selected values of $v(t)$, where t is measured in seconds and $v(t)$ is measured in meters per second, are given in the table below.

t (seconds)	0	4	10	15	19	19.9	19.99	19.999	19.9999
$v(t)$ (meters per second)	0	50	75	87	93	94.85	94.985	94.9984	94.9997

a) Use the data in the table to estimate the velocity of the airplane at takeoff, that is, estimate the value of $\lim_{t \to 20^-} v(t)$.

b) Use the data in the table to estimate the value of $v'(6)$. Using correct units, interpret the meaning of the value in the context of the problem.

c) Approximate the value of the definite integral $\int_0^{15} v(t)dt$ using a right Riemann sum with three subintervals indicated in the table. Using correct units, explain the meaning of the definite integral $\int_0^{15} v(t)dt$ in the context of the problem.

d) Suppose the velocity of a second airplane can be modeled by $f(t) = 15.7 \cdot \ln(t^2 + t + 1)$ for $0 \le t \le 20$,seconds. Using this model, find the acceleration of the plane at the time it reaches a velocity of 58 meters/sec (the stall speed).

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Answers and Rubrics (AB)

Answers to Multiple-Choice Questions

11	C
12	D
13	B
14	C
15	A
16	C
17	B
18	B
19	C
20	D

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Rubrics for Free-Response Questions

Question 3

Solutions	Point Allocation
(A) $g(4) = 5 + \int_2^4 f(t)\,dt = 5 + \left(-\frac{1}{2}\right) + 2 = \frac{13}{2}$	1 : answer
(B) $g'(x) = f(x)$ The function g is increasing on the intervals $-1 \le x \le 1$ and $3 \le x \le 4$ because $g' = f$ is nonnegative on these intervals.	$2 : \begin{cases} 1 : \text{answer} \\ 1 : \text{justification} \end{cases}$
(C) $g'(x) = f(x) = 0 \Rightarrow x = 1, x = 3$ $\begin{array}{c\|c} x & g(x) \\ \hline -1 & \frac{7}{2} \\ 1 & \frac{11}{2} \\ 3 & \frac{9}{2} \\ 4 & \frac{13}{2} \end{array}$ The absolute minimum value of g is $\frac{7}{2}$, and the absolute maximum value of g is $\frac{13}{2}$.	$4 : \begin{cases} 1 : \text{considers } g'(x) = 0 \\ 1 : \text{identifies } x = 1 \text{ and } x = 3 \text{ as candidates} \\ 1 : \text{answer} \\ 1 : \text{justification} \end{cases}$
(D) $h'(x) = 1 \cdot g(x) + x \cdot g'(x)$ $h'(2) = 1 \cdot g(2) + 2 \cdot g'(2) = 1(5) + 2(-1) = 3$	$2 : \begin{cases} 1 : h'(x) \\ 1 : h'(2) \end{cases}$

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4.

a) $\lim_{t \to 20^-} v(t) = 95$ meters / sec	1 : estimate
b) $v'(6) \approx \frac{v(10) - v(4)}{10 - 4} = \frac{75 - 50}{6} = 4.167$ (or 4.166) The velocity of the plane is increasing at a rate of approximately 4.167 (or 4.166) meters / sec^2 at time $t = 6$ seconds.	2 : $\begin{cases} 1: \text{estimate} \\ 1: \text{interpretation with units} \end{cases}$
c) $\int_0^{15} v(t)dt \approx 4 \cdot v(4) + 6 \cdot v(10) + 5 \cdot v(15)$ $= 4 \cdot 50 + 6 \cdot 75 + 5 \cdot 87$ $= 1085$ meters. $\int_0^{15} v(t)dt$ is the total distance traveled by the airplane on the runway, in meters, over the time interval $0 \le t \le 15$ seconds.	3 : $\begin{cases} 1: \text{right Riemann sum} \\ 1: \text{approximation} \\ 1: \text{explanation} \end{cases}$
d) $f(t) = 15.7 \cdot \ln(t^2 + t + 1) = 58 \Rightarrow t = 5.782205$ The airplane reaches a velocity of 58 meters / sec at time $a = 5.782205$ seconds. The acceleration of the airplane at that time is $f'(a) = 4.905$ meters / sec^2	3 : $\begin{cases} 1: \text{sets } f(t) = 58 \\ 1: \text{uses } f'(t) \\ 1: \text{answer} \end{cases}$

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