Name: Date:

1B Exercises

Finding Limits Analytically

Complete the table and use the result to estimate the limit. Use a graphing utility to graph the function to confirm your result. Sketch the graph in the neighborhood of the limit point.

Find the limit.

19. $\lim_{x\to 1} \frac{x}{x^2+4} = \frac{1}{1^2+4} = \frac{1}{5}$

21. $\lim_{x\to 7} \frac{3x}{\sqrt{x+2}} = \frac{3(7)}{\sqrt{7+2}} = \frac{21}{3} = 7$

Use the information to evaluate the limits

39. $\lim_{x\to c} f(x) = 4$

(a) $\lim_{x\to c} [f(x)]^3$

(b) $\lim_{x\to c} \sqrt{f(x)}$

(c) $\lim_{x\to c} [3f(x)]$

(d) $\lim_{x\to c} [f(x)]^{3/2}$

39. (a) $\lim_{x\to c}[f(x)]^3 = \left[\lim_{x\to c} f(x)\right]^3 = (4)^3 = 64$

(b) $\lim_{x\to c}\sqrt{f(x)} = \sqrt{\lim_{x\to c} f(x)} = \sqrt{4} = 2$

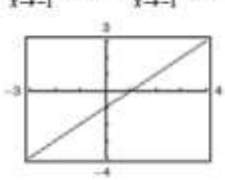
(c) $\lim_{x\to c}[3f(x)] = 3\lim_{x\to c} f(x) = 3(4) = 12$

(d) $\lim_{x\to c}[f(x)]^{3/2} = \left[\lim_{x\to c} f(x)\right]^{3/2} = (4)^{3/2} = 8$

Find the limit of the function (if it exists). Write a simpler function that agrees with the given function at all but one point. Use a graphing utility to confirm your result.

45. $f(x) = \frac{x^2-1}{x+1}$ and $g(x) = x - 1$ agree except at $x = -1$.

$\lim_{x\to -1} f(x) = \lim_{x\to -1} g(x) = -2$



46. $\lim_{x\to -1} \frac{2x^2-x-3}{x+1}$

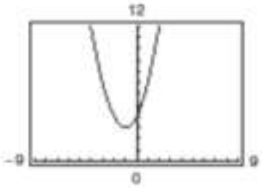
$$\frac{2x^2-x-3}{x+1} = \frac{(x+1)(2x-3)}{x+1} = \frac{2x-3}{x+1}$$

So, $f(x) = g(x) = 2x - 3$ for all points where $x \neq -1$

so $\lim_{x\to -1} f(x) = \lim_{x\to -1} g(x) = g(-1) = -5$

47. $f(x) = \dfrac{x^3 - 8}{x - 2}$ and $g(x) = x^2 + 2x + 4$ agree except at $x = 2$.

$\lim_{x\to 2} f(x) = \lim_{x\to 2} g(x) = 12$



49. $\displaystyle \lim_{x\to 0}\frac{x}{x^2 - x} = \lim_{x\to 0}\frac{x}{x(x-1)} = \lim_{x\to 0}\frac{1}{x-1} = -1$

52. $\displaystyle \lim_{x\to 3}\frac{3 - x}{x^2 - 9}$

To be checked on Assignement

53. $\displaystyle \lim_{x\to -3}\frac{x^2 + x - 6}{x^2 - 9} = \lim_{x\to -3}\frac{(x+3)(x-2)}{(x+3)(x-3)}$

$\displaystyle = \lim_{x\to -3}\frac{x-2}{x-3} = \frac{-5}{-6} = \frac{5}{6}$

55. $\displaystyle \lim_{x\to 4}\frac{\sqrt{x+5} - 3}{x - 4} = \lim_{x\to 4}\frac{\sqrt{x+5} - 3}{x - 4}\cdot\frac{\sqrt{x+5} + 3}{\sqrt{x+5} + 3}$

$\displaystyle = \lim_{x\to 4}\frac{(x+5) - 9}{(x-4)\left(\sqrt{x+5} + 3\right)} = \lim_{x\to 4}\frac{1}{\sqrt{x+5} + 3} = \frac{1}{\sqrt{9} + 3} = \frac{1}{6}$

58. $\displaystyle \lim_{x\to 0}\frac{\sqrt{2+x} - \sqrt{2}}{x}$

To be checked on Assignement

59. $\displaystyle \lim_{x\to 0}\frac{\frac{1}{3+x} - \frac{1}{3}}{x} = \lim_{x\to 0}\frac{3 - (3+x)}{(3+x)3(x)} = \lim_{x\to 0}\frac{-x}{(3+x)(3)(x)} = \lim_{x\to 0}\frac{-1}{(3+x)3} = -\frac{1}{9}$

60. $\displaystyle \lim_{x\to 0}\frac{[1/(x+4)] - (1/4)}{x}$

To be checked on Assignement

Determine the limit of the trigonometric function (if it exists).

65. $\lim_{x \to 0} \frac{\sin x}{5x} = \lim_{x \to 0} \left[\left(\frac{\sin x}{x}\right)\left(\frac{1}{5}\right)\right] = (1)\left(\frac{1}{5}\right) = \frac{1}{5}$

66. $\lim_{x \to 0} \frac{3(1 - \cos x)}{x}$

$$\lim_{x \to 0} 3\left(\frac{1 - \cos x}{x}\right) = 3(0) = 0$$

68. $\lim_{\theta \to 0} \frac{\cos \theta \tan \theta}{\theta}$

$$\lim_{\theta \to 0}\left(\frac{\cos \theta}{\theta}\right)\left(\frac{\sin \theta}{\cos \theta}\right) = \lim_{\theta \to 0} \frac{\sin \theta}{\theta} = 1$$

71. $\lim_{h \to 0} \frac{(1 - \cos h)^2}{h} = \lim_{h \to 0}\left[\frac{1 - \cos h}{h}(1 - \cos h)\right]$
$= (0)(0) = 0$

73. $\lim_{x \to \pi/2} \frac{\cos x}{\cot x} = \lim_{x \to \pi/2} \sin x = 1$

74. $\lim_{x \to \pi/4} \frac{1 - \tan x}{\sin x - \cos x}$

$$\lim_{x \to \frac{\pi}{4}}\left(\frac{1 - \tan x}{\sin x - \cos x}\right)\left(\frac{\cos x}{\cos x}\right)$$
$$= \lim_{x \to \frac{\pi}{4}} \frac{\cos x - \cos x \tan x}{(\sin x - \cos x)(\cos x)}$$
$$= \lim_{x \to \frac{\pi}{4}} \frac{\cos x - \sin x}{(\sin x - \cos x)(\cos x)}$$
$$= \lim_{x \to \frac{\pi}{4}} \frac{(-1)(\sin x - \cos x)}{(\sin x - \cos x)(\cos x)} = \frac{-1}{\frac{\sqrt{2}}{2}} = -\frac{2}{\sqrt{2}} = -\sqrt{2}$$

76. $\lim_{x \to 0} \frac{\sin 2x}{\sin 3x}$ $\left[\textit{Hint}\text{: Find } \lim_{x \to 0}\left(\frac{2 \sin 2x}{2x}\right)\left(\frac{3x}{3 \sin 3x}\right).\right]$

$$\lim_{x \to 0} \frac{\sin 2x}{\sin 3x} = \lim_{x \to 0}\left(\frac{\sin 2x}{2x}\right)\left(\frac{3x}{\sin 3x}\right)\left(\frac{2x}{3x}\right) = (1)(1)\left(\frac{2}{3}\right) = \frac{2}{3}$$

125. ***Graphical Reasoning*** Consider $f(x) = \dfrac{\sec x - 1}{x^2}$.

(a) Find the domain of f.

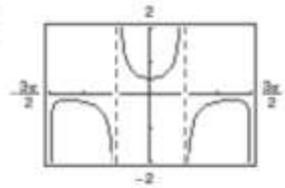
(b) Use a graphing utility to graph f. Is the domain of f obvious from the graph? If not, explain.

(c) Use the graph of f to approximate $\lim_{x \to 0} f(x)$.

(d) Confirm your answer to part (c) analytically.

125. $f(x) = \dfrac{\sec x - 1}{x^2}$

(a) The domain of f is all $x \neq 0, \pi/2 + n\pi$.

(b) 

The domain is not obvious. The hole at $x = 0$ is not apparent.

(c) $\lim_{x \to 0} f(x) = \dfrac{1}{2}$

(d) $$\frac{\sec x - 1}{x^2} = \frac{\sec x - 1}{x^2} \cdot \frac{\sec x + 1}{\sec x + 1} = \frac{\sec^2 x - 1}{x^2(\sec x + 1)}$$

$$= \frac{\tan^2 x}{x^2(\sec x + 1)} = \frac{1}{\cos^2 x}\left(\frac{\sin^2 x}{x^2}\right)\frac{1}{\sec x + 1}$$

So, $$\lim_{x \to 0} \frac{\sec x - 1}{x^2} = \lim_{x \to 0} \frac{1}{\cos^2 x}\left(\frac{\sin^2 x}{x^2}\right)\frac{1}{\sec x + 1}$$

$$= 1(1)\left(\frac{1}{2}\right) = \frac{1}{2}.$$