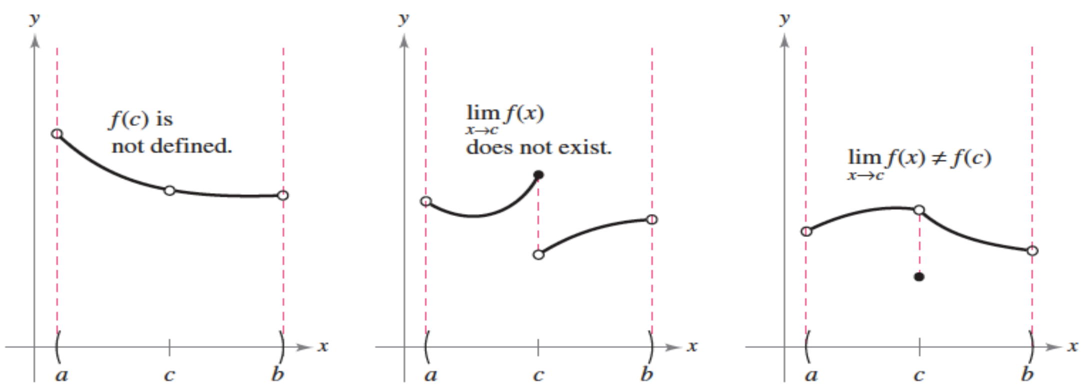
Calculus Name: Date:

1C: Continuity and Intermediate Value Theorem

Up to this point we have been investigating the nature of a function as it approaches a finite value. We will now take the concept of a limit a step further to ask if we can find the limit for any point on an interval. We call this *continuity*.

A function is **continuous** at a point $x = c$ if there is no break in the function at this point. When there is a break in the function, it can happen in three ways.



Three conditions exist for which the graph of f is not continuous at $x = c$.

If none of these conditions occur, then we say that the function is continuous at $x = c$.

Definition of Continuity

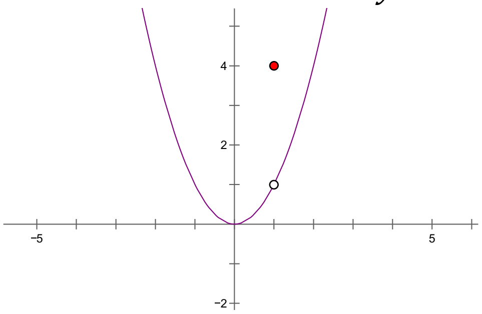
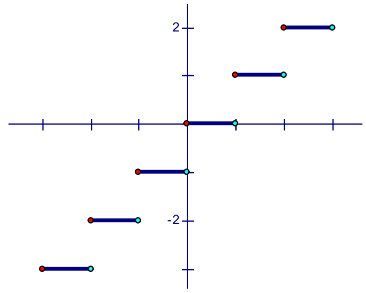
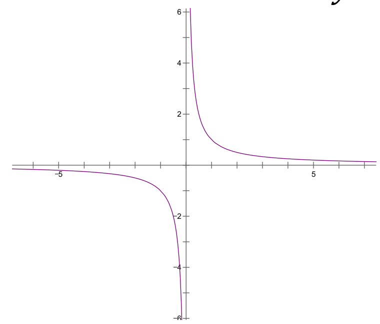
Continuity at a point: A function f is continuous at c if three conditions are met:

1. $f(c)$ is defined
2. $\lim_{x \to c} f(x)$ exists
3. $\lim_{x \to c} f(x) = f(c)$

Continuity on an open interval: A function f is continuous on an open interval (a, b) if it is continuous at every point on the interval. If it is continuous on the entire real line $(-\infty, \infty)$, then it is ***everywhere continuous.***

Discontinuities can be ***removable*** if the function can be redefined at $f(c)$ to be continuous (possibly by simplifying the function). If it can't be redefined, then it is ***nonremovable..***

Types of discontinuity

Removable discontinuity	*Jump discontinuity*	*Infinite discontinuity*
		
$\lim_{x \to 1} f(x) = 1$	$\lim_{x \to 1} f(x)$ Does Not Exist because it's not the same from both sides	$\lim_{x \to 0} f(x)$ Does Not Exist because it's not the same from both sides

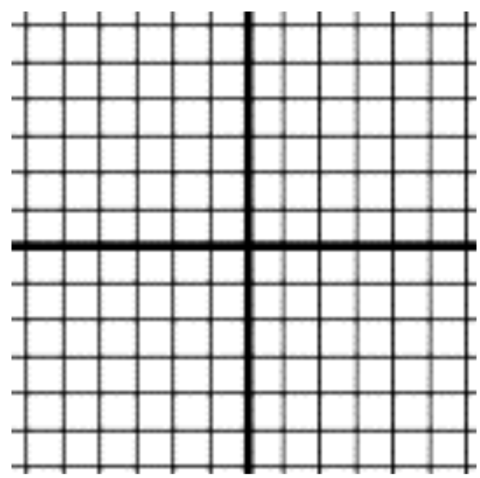
Try it! **Sketch a graph of the function, then find where there are the discontinuities and what kind they are?**

a) $f(x) = \frac{1}{x}$

b) $g(x) = \frac{x^2-1}{x-1}$

c) $h(x) = \begin{cases} x+1, & x \leq 0 \\ x^2+1, & x > 0 \end{cases}$

d) $j(x) = \csc(\mathrm{x})$



The Greatest Integer Function (a.k.a. the floor function)

The **Greatest Integer Function** takes a real number in the domain and returns *the greatest integer that is less than x .* We write this as $g(x) = [\![x]\!]$ or $\mathrm{g(x)} = \mathrm{floor(x)}$

Consider this:

a) Is $g(x) = [\![x]\!]$ continuous on $(0,1)$?

b) Find $\lim_{x \to 1^-} [\![x]\!] =$

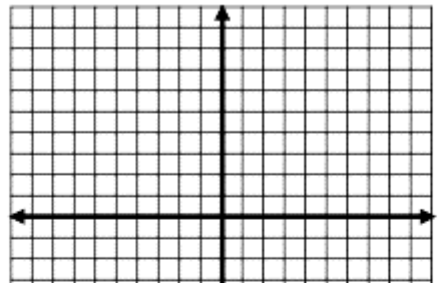
c) $\lim_{x \to 1^+} [\![x]\!] =$

d) Is $g(x) = [\![x]\!]$ continuous on $(0,2)$?

Continuity on a closed interval: A function f is continuous on a closed interval $[a, b]$ if it is continuous on (a, b) and

$$\lim_{x \to a^+} f(x) = f(a), \qquad and \lim_{x \to b^-} f(x) = f(b)$$

Example Discuss the continuity of $f(x) = \sqrt{16 - x^2}$
(That is, find the interval(s) on which it is continuous)



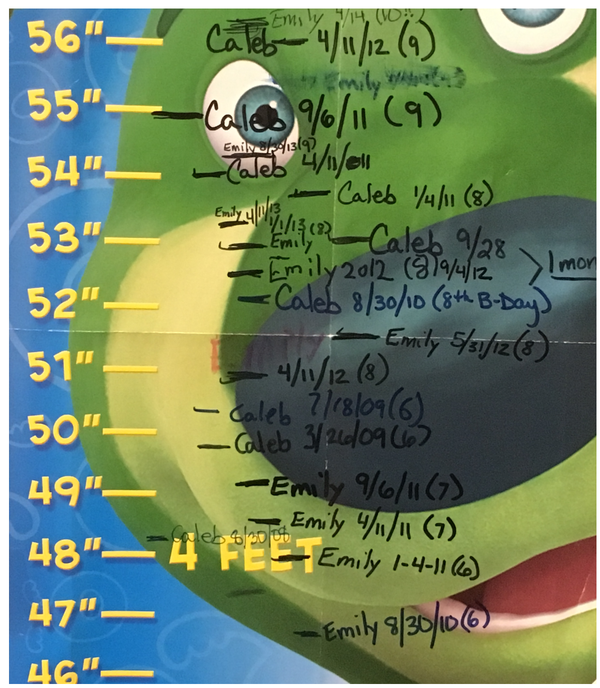
Intermediate Value Theorem (IVT)

The idea of continuity opens the door to a powerful theorem called the *intermediate value theorem.*

Consider these:

- One cold night in December, the lowest recorded temperature in Grants Pass got down to $-18.3°C$. If it later warmed up to $5°C$ that day, was the temperature ever $0°C$? Was the temperature ever $\pi°C$?

- My Daughter, Emily, was 46.5" tall on 8/30/10, and 48" tall on 1/4/11 (according to the wall height chart). Was she ever exactly 47.25" tall?



Why can you be confident in these answers?

Intermediate Value Theorem

If f is continuous on $[a, b]$ and $f(a) \neq f(b)$, and k is any number between $f(a)$ and $f(b)$, then there must be at least on value c in $[a, b]$ such that $f(c) = k$.

Example

1. A car travels on a straight track. During the time interval $0 \le t \le 60$ seconds, the car's velocity v, measured in feet per second, and acceleration a, measured in feet per second per second, are continuous functions. The table below shows selected values of these functions.

t (sec)	0	15	25	30	35	50	60
$v(t)$ (ft/sec)	−20	−30	−20	−14	−10	0	10
$a(t)$ (ft/sec^2)	1	5	2	1	2	4	2

For $0<t<60$, must there be a time t when $v(t)=-5$? Justify your answer.

2. The table below shows selected values of a continuous function f. For $0 \le x \le 13$, what is the fewest possible number of times $f(x) = 4$?

x	0	4	6	8	13
$f(x)$	3	4.5	3	2.5	4.4