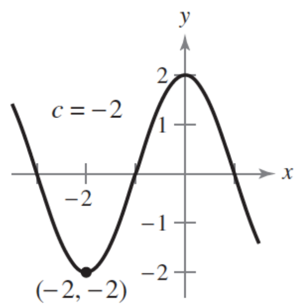
Name: Date:

1C Exercises

Continuity and the Intermediate Value Theorem

In Exercises 1–6, use the graph to determine the limit, and discuss the continuity of the function.

2.



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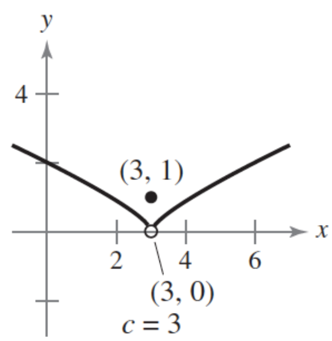

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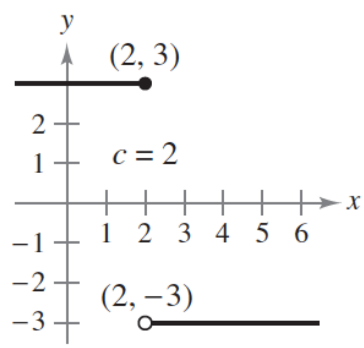


In Exercises 7–26, find the limit (if it exists). If it does not exist, explain why.

7. $\displaystyle\lim_{x\to 8^+} \frac{1}{x+8}$

9. $\displaystyle\lim_{x\to 5^+} \frac{x-5}{x^2-25}$

15. $\displaystyle\lim_{\Delta x\to 0^-} \frac{\dfrac{1}{x+\Delta x} - \dfrac{1}{x}}{\Delta x}$

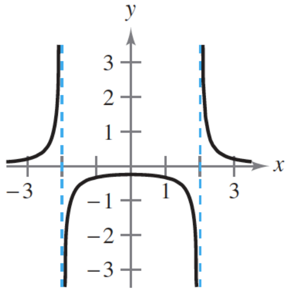
17. $\displaystyle\lim_{x\to 3^-} f(x)$, where $f(x) = \begin{cases} \dfrac{x+2}{2}, & x \le 3 \\ \dfrac{12-2x}{3}, & x > 3 \end{cases}$

18. $\displaystyle\lim_{x\to 2} f(x)$, where $f(x) = \begin{cases} x^2 - 4x + 6, & x < 2 \\ -x^2 + 4x - 2, & x \ge 2 \end{cases}$

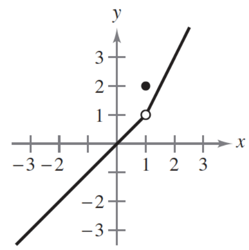
24. $\displaystyle\lim_{x\to 2^+} (2x - [\![x]\!])$

In Exercises 27–30, discuss the continuity of each function.

27. $f(x) = \dfrac{1}{x^2 - 4}$



30. $f(x) = \begin{cases} x, & x < 1 \\ 2, & x = 1 \\ 2x - 1, & x > 1 \end{cases}$



In Exercises 31–34, discuss the continuity of the function on the closed interval.

34. $g(x) = \dfrac{1}{x^2 - 4}$ $\quad [-1, 2]$

In Exercises 35– 60, find the -values (if any) at which is not continuous. Which of the discontinuities are removable?

41. $f(x) = 3x - \cos x$

47. $f(x) = \dfrac{x + 2}{x^2 - 3x - 10}$

54. $f(x) = \begin{cases} -2x, & x \leq 2 \\ x^2 - 4x + 1, & x > 2 \end{cases}$

Find the value of *a* and *b* that make the function continuous.

63. $f(x) = \begin{cases} 3x^2, & x \geq 1 \\ ax - 4, & x < 1 \end{cases}$

67. $f(x) = \begin{cases} 2, & x \leq -1 \\ ax + b, & -1 < x < 3 \\ -2, & x \geq 3 \end{cases}$

Writing In Exercises 83–86, explain why the function has a zero in the given interval.

83. $f(x) = \frac{1}{12}x^4 - x^3 + 4 \qquad [1, 2]$

86. $f(x) = -\dfrac{5}{x} + \tan\left(\dfrac{\pi x}{10}\right) \qquad [1, 4]$

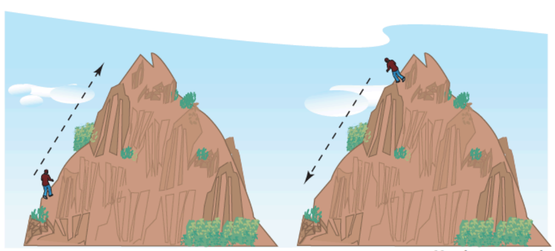
In Exercises 91–94, verify that the Intermediate Value Theorem applies to the indicated interval and find the value of guaranteed by the theorem.

91. $f(x) = x^2 + x - 1, \quad [0, 5], \quad f(c) = 11$

94. $f(x) = \dfrac{x^2 + x}{x - 1}, \quad \left[\dfrac{5}{2}, 4\right], \quad f(c) = 6$

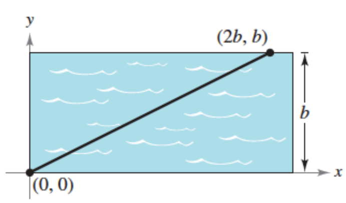
Challenge Problems (Optional)

107. ***Déjà Vu*** At 8:00 A.M. on Saturday a man begins running up the side of a mountain to his weekend campsite (see figure). On Sunday morning at 8:00 A.M. he runs back down the mountain. It takes him 20 minutes to run up, but only 10 minutes to run down. At some point on the way down, he realizes that he passed the same place at exactly the same time on Saturday. Prove that he is correct. [*Hint:* Let $s(t)$ and $r(t)$ be the position functions for the runs up and down, and apply the Intermediate Value Theorem to the function $f(t) = s(t) - r(t)$.]



Saturday 8:00 A.M. Sunday 8:00 A.M.

114. ***Creating Models*** A swimmer crosses a pool of width b by swimming in a straight line from $(0, 0)$ to $(2b, b)$. (See figure.)



(a) Let f be a function defined as the y-coordinate of the point on the long side of the pool that is nearest the swimmer at any given time during the swimmer's crossing of the pool. Determine the function f and sketch its graph. Is f continuous? Explain.

(b) Let g be the minimum distance between the swimmer and the long sides of the pool. Determine the function g and sketch its graph. Is g continuous? Explain.