

Name: Solutions

Infinite Limits (And applications)

1.
$$\lim_{x \to -\infty} \frac{3x^3 - 2x + 4}{x^3 + x^2 - 1} \cdot \frac{\frac{1}{x^3}}{\frac{1}{x^3}} = \lim_{x \to -\infty} \frac{3x^3}{x^3} = 3$$

3. $\lim_{x \to \infty} \frac{2x-3}{x^2+6} = \lim_{x \to \infty} \frac{2x}{x^2} = \lim_{x \to \infty} \frac{2}{x} = 0$

2.
$$\lim_{x \to \infty} \frac{4x^4 + 2x^3 - 1005}{(3x^2 + 2)^2} = \lim_{x \to \infty} \frac{4x^4}{9x^4} = \frac{4}{9}$$
4.
$$\lim_{x \to -\infty} \frac{15x^{301}}{x^{300} + 3x^{299}} \cdot = \lim_{x \to -\infty} \frac{15x^{301}}{x^{300}} = \lim_{x \to -\infty} 15x = -\infty$$

5. Use your calculator to find this limit numerically

$$\lim_{x \to \infty} \left(1 + \frac{1}{n} \right)^n = e \approx 2.71828$$

Here are some good limit practice problems to tie it all together!

6.

Here's a modified part of an example from the Free Response portion of the 2007 AP exam.

x	f(x)	g(x)
1	6	2
2	9	3
3	10	4
4	-1	6

The functions f and g are continuous for all real numbers. The table above gives values of the functions and their first derivatives at selected values of x. The function h is given by h(x) = f(g(x)) - 6.

Explain why there must be a value r for 1 < r < 3 such that h(r) = -5.

$$h(1) = f(g(1)) - 6 = f(2) - 6 = 9 - 6 = 3$$

$$h(3) = f(g(3)) - 6 = f(4) - 6 = -1 - 6 = -7$$

Since f and g are continuous, h(x) is continuous. Furthermore, since $h(3) \le -5 \le h(1)$, by IVT, there exists a value r in (1,3) such that h(r) = -5.

7.

If $f(x) = x^3 - x^2 + x$, show that there is a number x = c on some interval (a, b) such that f(c) = 10. Give the interval (a, b) in your answer.

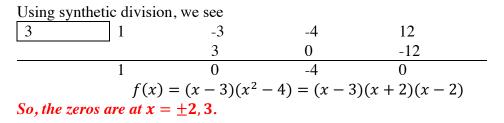
$$f(0) = 0, \qquad f(3) = 20$$

Since f(x) is continuous (all polynomials are continuous), and $f(0) \le 10 \le f(3)$, there exists a number x = c on the interval (0,3) such that f(c) = 10.

8. Given the two functions *f* and *h*, such that

$$f(x) = x^{3} - 3x^{2} - 4x + 12 \qquad \text{and} \qquad h(x) = \begin{cases} \frac{f(x)}{x - 3}, & x \neq 3\\ p, & x = 3 \end{cases}$$

(a) Find all the zeros of f.



(b) Find the value of p so that the function h is continuous at x = 3. Justify.

Now Consider $g(x) = \frac{f(x)}{x-3} = \frac{(x-3)(x-2)(x+2)}{x-3} = (x-2)(x+2)$ This give us g(3) = (3-2)(3+2) = 5. So, $\frac{f(x)}{x-3}$ has a removeable discontinuity at x = 3, and the g(3) = 5 for the function g(x) that is equal to f(x) at all points except at x = 3. *Therefore, if* p = 5, h(x) is continuous (because it will be equivalent to g(x) = (x-2)(x+2)).

9. For $f(x) = \begin{cases} cx^2 - 3, & x \le 2 \\ cx + 2, & x > 2 \end{cases}$, find the value of *c* to make *f* continuous at x = 2.

If
$$f(x)$$
 is continuous at $x = 2$,
 $c(2)^2 - 3 = c(2) + 2$
 $4c - 3 = 2c + 2$
 $2c = 5$
 $c = \frac{5}{2}$

10.
If
$$f(x) = \begin{cases} \frac{\sqrt{2x+5} - \sqrt{x+7}}{x-2}, & x \neq 2\\ k, & x = 2 \end{cases}$$
, and if f is continuous at $x = 2$, then $k = x = 2$.

We need to find the removable discontinuity at
$$x = 2$$

$$\frac{\sqrt{2x+5} - \sqrt{x+7}}{x-2} \cdot \frac{\sqrt{2x+5} + \sqrt{x+7}}{\sqrt{2x+5} + \sqrt{x+7}}$$

$$= \frac{((2x+5) - (x+7))}{(x-2)(\sqrt{2x+5} + \sqrt{x+7})}$$

$$= \frac{x-2}{(x-2)(\sqrt{2x+5} + \sqrt{x+7})}$$

$$= \frac{1}{(\sqrt{2x+5} + \sqrt{x+7})}$$
at $x = 2$, we have

$$\frac{1}{\left(\sqrt{2(2)+5}+\sqrt{(2)+7}\right)} = \frac{1}{6}$$

So, $k = \frac{1}{6}$