1D Exercises

Infinite Limits (And applications)

1. $\lim_{x\to-\infty} \frac{3x^3-2x+4}{x^3+x^2-1} \cdot \frac{\frac{1}{x^3}}{\frac{1}{x^3}}$

$$= \lim_{x\to-\infty} \frac{3x^3}{x^3} = 3$$

2. $\lim_{x\to\infty} \frac{4x^4+2x^3-1005}{(3x^2+2)^2}$

$$= \lim_{x\to\infty} \frac{4x^4}{9x^4}$$
$$= \frac{4}{9}$$

3. $\lim_{x\to\infty} \frac{2x-3}{x^2+6}$

$$= \lim_{x\to\infty} \frac{2x}{x^2} = \lim_{x\to\infty} \frac{2}{x} = 0$$

4. $\lim_{x\to-\infty} \frac{15x^{301}}{x^{300}+3x^{299}} \cdot$

$$= \lim_{x\to-\infty} \frac{15x^{301}}{x^{300}} = \lim_{x\to-\infty} 15x = -\infty$$

5. Use your calculator to find this limit numerically

$$\lim_{x\to\infty} \left(1+\frac{1}{n}\right)^n = e \approx 2.71828$$

Here are some good limit practice problems to tie it all together!

6. Here's a modified part of an example from the Free Response portion of the 2007 AP exam.

x	$f(x)$	$g(x)$
1	6	2
2	9	3
3	10	4
4	-1	6

The functions f and g are continuous for all real numbers. The table above gives values of the functions and their first derivatives at selected values of x. The function h is given by $h(x) = f(g(x)) - 6$. Explain why there must be a value r for $1 < r < 3$ such that $h(r) = -5$.

$$h(1) = f(g(1)) - 6 = f(2) - 6 = 9 - 6 = 3$$
$$h(3) = f(g(3)) - 6 = f(4) - 6 = -1 - 6 = -7$$

Since f and g are continuous, $h(x)$ is continuous. Furthermore, since $h(3) \le -5 \le h(1)$, by IVT, there exists a value r in $(1,3)$ such that $h(r) = -5$.

7. If $f(x) = x^3 - x^2 + x$, show that there is a number $x = c$ on some interval (a,b) such that $f(c) = 10$. Give the interval (a,b) in your answer.

$$f(0) = 0, \qquad f(3) = 20$$

*Since $\boldsymbol{f(x)}$ **is continuous** (all polynomials are continuous), and $f(0) \le 10 \le f(3)$, there exists a number $x = c$ on the interval $(0,3)$ such that $f(c) = 10$.*

8. Given the two functions f and h, such that

$$f(x) = x^3 - 3x^2 - 4x + 12 \qquad \text{and} \qquad h(x) = \begin{cases} \dfrac{f(x)}{x-3}, & x \neq 3 \\ p, & x = 3 \end{cases}$$

(a) Find all the zeros of f.

Using synthetic division, we see

$\boxed{3}$	1	-3	-4	12
		3	0	-12
	1	0	-4	0

$$f(x) = (x-3)(x^2-4) = (x-3)(x+2)(x-2)$$

So, the zeros are at $\boldsymbol{x = \pm 2, 3}$.

(b) Find the value of p so that the function h is continuous at $x = 3$. Justify.

Now Consider $g(x) = \frac{f(x)}{x-3} = \frac{(x-3)(x-2)(x+2)}{x-3} = (x-2)(x+2)$

This give us $g(3) = (3-2)(3+2) = 5$. So, $\frac{f(x)}{x-3}$ has a removeable discontinuity at $x = 3$, and the $g(3) = 5$ for the function $g(x)$ that is equal to $f(x)$ at all points except at $x = 3$. ***Therefore, if $\boldsymbol{p = 5}$,*** $h(x)$ is continuous (because it will be equivalent to $g(x) = (x-2)(x+2)$).

9. For $f(x) = \begin{cases} cx^2 - 3, & x \le 2 \\ cx + 2, & x > 2 \end{cases}$, find the value of c to make f continuous at $x = 2$.

If $f(x)$ is continuous at $x = 2$,

$$c(2)^2 - 3 = c(2) + 2$$
$$4c - 3 = 2c + 2$$
$$2c = 5$$
$$c = \frac{5}{2}$$

10.

If $f(x)=\begin{cases}\dfrac{\sqrt{2x+5}-\sqrt{x+7}}{x-2}, & x\neq 2\\ k, & x=2\end{cases}$, and if f is continuous at $x=2$, then $k=$

We need to find the removable discontinuity at $x=2$

$$\frac{\sqrt{2x+5}-\sqrt{x+7}}{x-2}\cdot\frac{\sqrt{2x+5}+\sqrt{x+7}}{\sqrt{2x+5}+\sqrt{x+7}}$$
$$=\frac{\left((2x+5)-(x+7)\right)}{(x-2)\left(\sqrt{2x+5}+\sqrt{x+7}\right)}$$
$$=\frac{x-2}{(x-2)\left(\sqrt{2x+5}+\sqrt{x+7}\right)}$$
$$=\frac{1}{\left(\sqrt{2x+5}+\sqrt{x+7}\right)}$$

at $x=2$, we have

$$\frac{1}{\left(\sqrt{2(2)+5}+\sqrt{(2)+7}\right)}=\frac{1}{6}$$

So, $\boldsymbol{k=\frac{1}{6}}$