2C Exercises

Product and Quotient Rule

Find the derivative of the function.

25. $f(x) = \dfrac{4 - 3x - x^2}{x^2 - 1}$

$$f(x) = \frac{4 - 3x - x^2}{x^2 - 1}$$

$$f'(x) = \frac{(x^2 - 1)(-3 - 2x) - (4 - 3x - x^2)(2x)}{(x^2 - 1)^2}$$

$$= \frac{-3x^2 + 3 - 2x^3 + 2x - 8x + 6x^2 + 2x^3}{(x^2 - 1)^2}$$

$$= \frac{3x^2 - 6x + 3}{(x^2 - 1)^2}$$

$$= \frac{3(x^2 - 2x + 1)}{(x^2 - 1)^2}$$

$$= \frac{3(x - 1)^2}{(x - 1)^2(x + 1)^2} = \frac{3}{(x + 1)^2}, x \neq 1$$

27. $f(x) = x\left(1 - \dfrac{4}{x + 3}\right)$

$$f(x) = x\left(1 - \frac{4}{x + 3}\right) = x - \frac{4x}{x + 3}$$

$$f'(x) = 1 - \frac{(x + 3)4 - 4x(1)}{(x + 3)^2}$$

$$= \frac{(x^2 + 6x + 9) - 12}{(x + 3)^2}$$

$$= \frac{x^2 + 6x - 3}{(x + 3)^2}$$

29. $f(x) = \dfrac{3x - 1}{\sqrt{x}}$

$$f(x) = \frac{3x - 1}{\sqrt{x}} = 3x^{1/2} - x^{-1/2}$$

$$f'(x) = \frac{3}{2}x^{-1/2} + \frac{1}{2}x^{-3/2}$$

$$= \frac{3x + 1}{2x^{3/2}}$$

Alternate solution:

$$f(x) = \frac{3x - 1}{\sqrt{x}} = \frac{3x - 1}{x^{1/2}}$$

$$f'(x) = \frac{x^{1/2}(3) - (3x - 1)\left(\frac{1}{2}\right)\left(x^{-1/2}\right)}{x}$$

$$= \frac{\frac{1}{2}x^{-1/2}(3x + 1)}{x}$$

$$= \frac{3x + 1}{2x^{3/2}}$$

33. $f(x) = \dfrac{2 - \dfrac{1}{x}}{x - 3}$

$$f(x) = \frac{2 - (1/x)}{x - 3} = \frac{2x - 1}{x(x - 3)} = \frac{2x - 1}{x^2 - 3x}$$

$$f'(x) = \frac{(x^2 - 3x)2 - (2x - 1)(2x - 3)}{(x^2 - 3x)^2} = \frac{2x^2 - 6x - 4x^2}{(x^2 - 3x}$$

$$= \frac{-2x^2 + 2x - 3}{(x^2 - 3x)^2} = -\frac{2x^2 - 2x + 3}{x^2(x - 3)^2}$$

35. $f(x) = (2x^3 + 5x)(x - 3)(x + 2)$

$$f(x) = (2x^3 + 5x)(x - 3)(x + 2)$$

$$f'(x) = (6x^2 + 5)(x - 3)(x + 2) + (2x^3 + 5x)(1)(x + 2) + (2x^3 + 5x)(x - 3)(1)$$

$$= (6x^2 + 5)(x^2 - x - 6) + (2x^3 + 5x)(x + 2) + (2x^3 + 5x)(x - 3)$$

$$= (6x^4 + 5x^2 - 6x^3 - 5x - 36x^2 - 30) + (2x^4 + 4x^3 + 5x^2 + 10x) + (2x^4 + 5x^2 - 6x^3 - 15x)$$

$$= 10x^4 - 8x^3 - 21x^2 - 10x - 30$$

Note: You could simplify first:

$$f(x) = (2x^3 + 5x)(x^2 - x - 6)$$

37. $f(x) = \dfrac{x^2 + c^2}{x^2 - c^2}$, c is a constant

$$f(x) = \frac{x^2 + c^2}{x^2 - c^2}$$

$$f'(x) = \frac{(x^2 - c^2)(2x) - (x^2 + c^2)(2x)}{(x^2 - c^2)^2}$$

$$= -\frac{4xc^2}{(x^2 - c^2)^2}$$

Write the equation of the tangent lines through the given point.

63. $f(x) = (x^3 + 4x - 1)(x - 2)$, $(1, -4)$

66. $f(x) = \dfrac{(x - 1)}{(x + 1)}$, $\left(2, \dfrac{1}{3}\right)$

(a) $f(x) = (x^3 + 4x - 1)(x - 2)$, $(1, -4)$

$$f'(x) = (x^3 + 4x - 1)(1) + (x - 2)(3x^2 + 4)$$

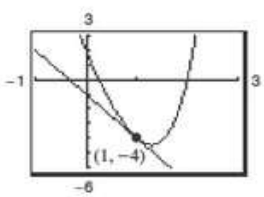
$$= x^3 + 4x - 1 + 3x^3 - 6x^2 + 4x - 8$$

$$= 4x^3 - 6x^2 + 8x - 9$$

$f'(1) = -3$; Slope at $(1, -4)$

Tangent line: $y + 4 = -3(x - 1) \Rightarrow y = -3x - 1$

(b)



(c) Graphing utility confirms $\dfrac{dy}{dx} = -3$ at $(1, -4)$.

83. *Area* The length of a rectangle is given by $6t + 5$ and its height is $\sqrt{t}$, where t is time in seconds and the dimensions are in centimeters. Find the rate of change of the area with respect to time.

$$\text{Area} = A(t) = (6t + 5)\sqrt{t} = 6t^{3/2} + 5t^{1/2}$$

$$A'(t) = 9t^{1/2} + \frac{5}{2}t^{-1/2} = \frac{18t + 5}{2\sqrt{t}} \text{ cm}^2/\text{sec}$$

87. *Population Growth* A population of 500 bacteria is introduced into a culture and grows in number according to the equation

$$P(t) = 500\left(1 + \frac{4t}{50 + t^2}\right)$$

where t is measured in hours. Find the rate at which the population is growing when $t = 2$.

$$P(t) = 500\left[1 + \frac{4t}{50 + t^2}\right]$$

$$P'(t) = 500\left[\frac{(50 + t^2)(4) - (4t)(2t)}{(50 + t^2)^2}\right] = 500\left[\frac{200 - 4t^2}{(50 + t^2)^2}\right] = 2000\left[\frac{50 - t^2}{(50 + t^2)^2}\right]$$

$$P'(2) \approx 31.55 \text{ bacteria/h}$$

Find the second derivative

97. $f(x) = \dfrac{x}{x - 1}$

$$f(x) = \frac{x}{x - 1}$$

$$f'(x) = \frac{(x - 1)(1) - x(1)}{(x - 1)^2} = \frac{-1}{(x - 1)^2}$$

$$f''(x) = \frac{2}{(x - 1)^3}$$

99. $f(x) = x \sin x$

$$f(x) = x \sin x$$

$$f'(x) = x \cos x + \sin x$$

$$f''(x) = x(-\sin x) + \cos x + \cos x$$

$$= -x \sin x + 2 \cos x$$

AP Practice...

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
-1	3	4	-2	2
0	2	-3	5	-1

The table above gives values for two differentiable functions and their derivatives at selected values of x. Use the table to evaluate the following.

(a) $h'(0)$ if $h(x)=\dfrac{f(x)}{g(x)}$

(b) $h'(-1)$ if $h(x)=x\cdot f(x)\cdot g(x)$

$$h'(0) = \frac{f'(0)g(0) - f(0)g'(0)}{g(0)^2}$$
$$= \frac{(-3)(5) - (2)(-1)}{5^2}$$
$$= -\mathbf{\frac{12}{25}}$$

$$h'(-1) = (-1)\big(f'(-1)g(-1) + f(-1)g'(-1)\big)$$
$$= -\big((4)(-2) + (3)(2)\big) = \mathbf{5}$$