Name: Date:

2D: The Chain Rule

Our goal is to find a method for finding the derivative of any function. So, now we turn to the composition of functions. For example, we can take the functions $f(x) = x^2$ and $g(x) = \sin x$ and we can compose them in two different ways:

$$y = \sin(x^2) \quad , \qquad \textit{and} \qquad y = (\sin x)^2$$

Notice, these are not sums, products, or quotients. So, we need something new… we need **the chain rule**!

Leibniz Notation and the Chain Rule

The best and easiest way to see the chain rule is using Leibniz's notation, $\frac{dy}{dx}$. Suppose that we can write the function y as a function of u and u as a function of

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

The Chain Rule

If $f(u)$ is a differentiable function of u and $u = g(x)$ is a differentiable function of x, then $y = f\big(g(x)\big)$ is differentiable, and ‘

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

Break it Up!

$y = f\big(g(x)\big)$	$u = g(x)$	$y = f(u)$
$y = (x^5 + x)^{10}$		
$y = \sqrt{3x^2 + x}$		
$y = \sin 2x$		
$y = \tan^2 x$		

Try it Out Use the chain rule to find $\frac{dy}{dx}$

(a) $y = (x^5 + x)^{10}$

(b) $y = \sqrt{3x^2 + x}$

Examples continued: Find $\frac{dy}{dx}$

(c) $y = \sin 2x$

(d) $y = \tan^2 x$

(e) $y = \frac{1}{x+1}$

(f) $y = x \sec(x^2 + x + 1)$

(g) $y = \sec(4 - \cos^3 5x)$

(h) $y = \left(\frac{3x-1}{x^2+3}\right)^2$