Calculus Name: Date:

2E: Implicit Differentiation

When a function is written as an equation where y is defined by the dependent value x, we say that this is an **explicit function** of x. For example, $y = \frac{1}{x}$ is an implicit function. However, sometimes we have equations that have bot y and x on the same side of the equation, such as $xy = 1$. We call these equations **implicit functions** that are not solved for the dependent variable.

- **Explicit function**: $y = x^2 + 3x - 2$, $y = x\sin(2x + \pi)$, $y = \frac{x+3}{x^3+3x-1}$
- **Implicit function:** $x^2 + y^2 = 25$, $y^2 - x^3 + 2x = 1$ $x\sin y + y\cos x = 4\pi$

To find the derivatives, we could try to solve these equations for y, but usually this would just make a big mess and often be impossible. However, the chain rule provides the key to differentiating these functions if we remember that *y is a function of x*.

Explore: Find the $\frac{dy}{dx}$ of these equivalent functions first without the chain rule, then with it.

No chain rule needed	*Use chain rule.*
$y = x + 1$	$y - x = 1$
$y = \frac{1}{x}$	$xy = 1$
$y = x^2$	$\frac{y}{x^2} = 1$

Example Find dy/dx as a function of x and y.

$$x^2 + y^2 = 25$$

Steps for Implicit Differentiation:

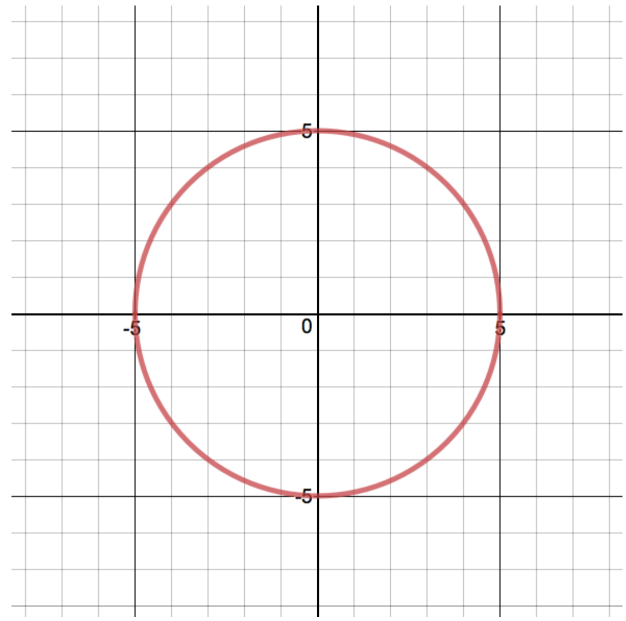
1. Take $\frac{d}{dx}$ of both sides of the equation.
2. Evaluate using the derivative rules
3. Solve for $\frac{dy}{dx}$.

Finding Tangent Lines :

Use implicit differentiation to find $\frac{dy}{dx}$, then find the equation of the graph at the given point.

Then find an equation of the tangent line at the given point and use Desmos to check your answer.

a) $x^2 + y^2 = 25$, at (3,4)



Draw the tangent line on the graph to the right.

b) $y^2 - x^3 + 2x = 1$, at (0,1)

c) $x \sin y + y \cos x = 4\pi$, at a point that **you** find (find the coordinates of a point on the graph first).

Second Derivatives! Things really get "exciting" when we find the 2nd derivative implicitly.

Find $\frac{d^2y}{dx^2}$ as a function of **x and y** if $2x^3 - 3y^2 = 8$