Calculus Name: Date:

2F: Derivatives of Logarithms and Exponentials

We almost have figure out the derivatives of all our fundamental functions. One of the last functions we need to work with is the derivative of exponential functions.

Here's a curious thought… So far, we can get derivatives of all powers of x except for x^{-1}

$\frac{dy}{dx} = 2x^1$	$\frac{dy}{dx} = x^0$	$\frac{dy}{dx} = x^{-1}$	$\frac{dy}{dx} = -1x^{-2}$	$\frac{dy}{dx} = -2x^{-3}$
$y = x^2$	$y = x^1$	$y = ?$	$y = x^{-1}$	$y = x^{-2}$

Hmmm, I wonder what the missing function is?

We might consider some functions we haven't found the derivative of yet like $y = a^x, y = e^x$, $y = \log_a x$, and $y = \ln x$. Let's try the natural log… just for kicks!

Use TI-Nspire to find these derivatives using tangent lines.

x	1	2	3	4	5	10	.1
$f'(x)$							

Do you recognize the values in $f'(x)$? Do you see a patterns?

Some Log Properties to Remember

So, we're going to be using logarithms. Here are some properties to remember:

• $\log_b mn = \log_b m + \log_b n$ • $\log_b \frac{m}{n} = \log_b m - \log_b n$ • $\log_b m^a = a \log_b m$ • $\log_e x = \ln x$	• $\log_b m = \frac{\log_a m}{\log_a b} = \frac{\ln m}{\ln b}$ • $\log_b b^x = x = b^{\log_b x}$ • $\log_b 1 = 0$

Derivative of $y = \ln x$ and $y = \log_b x$

Special Rule # 4: Derivative of Natural Logarithms

$$\frac{d}{dx}[\ln x] = \frac{1}{x}, \qquad \frac{d}{du}[\ln u] = \frac{u'}{u}$$

<u>Examples</u> Find the derivatives of these functions

a) $y = \cos(\ln x)$

b) $y = \ln(1 + \ln x)$

c) $h(t) = \ln \frac{\sqrt{t+2}}{t}$

Now we can generalize this using the change of base formula that says

$$\log_b m = \frac{\log_a m}{\log_a b} = \frac{\ln m}{\ln b}$$

So....

$$\frac{d}{dx}[\log_b x] =$$

Special Rule #5: Derivatives of General Logarithms

For any integer base $a > 0$,

$$\frac{d}{dx}[\log_b x] = \frac{1}{x \ln\ b}$$

$$\frac{d}{dx}[\log_b |x|] = \frac{1}{x \ln b}$$

$$\frac{d}{dx}[\log_b u] = \frac{u'}{u \ln b} \quad (chain\ rule)$$

Try it!

Find the derivative of $f(x) = \log_7 \frac{\sqrt{2x+3}}{\sin x}$

Logarithmic Differentiation

Sometimes it's a lot easier to take the derivative of the log of a function than to attack the function directly.

Steps for Logarithmic Differentiation:

1. Take the natural log of both sides
2. Find the derivative of both sides (Implicitly)
3. Solve for $\frac{dy}{dx}$ by multiplying by y and substitute original function.

<u>Examples</u> Find the derivatives

a) $y = \frac{e^x(x^2+4)^4}{\sqrt{x+5}(x^2+3)^3}$

b) $\frac{d}{dx}\left[(\sin x)^{\cos x}\right]$

c) $u = e^x$

Derivative of e^x and a^x

How's that for an easy one! But this is an amazingly powerful and special derivative. $f(x) = e^x$ is the **one and only function whose derivative is itself**!!

Specific Rule #6: Derivative of e^x

$\frac{d}{dx}[e^x] = e^x, \qquad \frac{d}{dx}[e^u] = u'e\text{^}u$

Try it. Find the derivatives:

$y = e^{x^2+x-3}$

$y = \sin(e^x)$

<u>How about different bases?</u>

Find the derivatives of these functions:

a) $y = 2^x$

Remember....

$$a = e^{lna}$$

b) $y = 4^{\cos x}$

Specific Rule #7: Derivative of $\boldsymbol{a}$

$$\frac{d}{dx}[a^x] = \ln a\,(a^x)$$