Calculus

Name: Date:

2F.1 Exercises - Solutions

Derivatives of Logarithms

Find the Derivatives (from 5.1)

47. $f(x) = \ln(3x)$

$$f(x) = \ln(3x)$$

$$f'(x) = \frac{1}{3x}(3) = \frac{1}{x}$$

49. $g(x) = \ln x^2$

$$g(x) = \ln x^2 = 2\ln x$$

$$g'(x) = \frac{2}{x}$$

53. $y = \ln(t+1)^2$

$$y = \ln(t+1)^2 = 2\ln(t+1)$$

$$y' = 2\frac{1}{t+1} = \frac{2}{t+1}$$

55. $y = \ln\left(x\sqrt{x^2-1}\right)$

$$y = \ln\left[x\sqrt{x^2-1}\right] = \ln x + \frac{1}{2}\ln\left(x^2-1\right)$$

$$\frac{dy}{dx} = \frac{1}{x} + \frac{1}{2}\left(\frac{2x}{x^2-1}\right) = \frac{2x^2-1}{x\left(x^2-1\right)}$$

57. $f(x) = \ln\left(\frac{x}{x^2+1}\right)$

$$f(x) = \ln\frac{x}{x^2+1} = \ln x - \ln\left(x^2+1\right)$$

$$f'(x) = \frac{1}{x} - \frac{2x}{x^2+1} = \frac{1-x^2}{x\left(x^2+1\right)}$$

63. $y = \ln\sqrt{\frac{x+1}{x-1}}$

$$y = \ln\sqrt{\frac{x+1}{x-1}} = \frac{1}{2}\left[\ln(x+1) - \ln(x-1)\right]$$

$$\frac{dy}{dx} = \frac{1}{2}\left[\frac{1}{x+1} - \frac{1}{x-1}\right] = \frac{1}{1-x^2}$$

71. $y = \ln\left|\frac{\cos x}{\cos x - 1}\right|$

$$y = \ln\left|\frac{\cos x}{\cos x - 1}\right|$$

$$= \ln|\cos x| - \ln|\cos x - 1|$$

$$\frac{dy}{dx} = \frac{-\sin x}{\cos x} - \frac{-\sin x}{\cos x - 1}$$

$$= -\tan x + \frac{\sin x}{\cos x - 1}$$

74. $y = \ln\sqrt{2 + \cos^2 x}$

Use implicit differentiation to find an equation of the tangent line to the graph at the given point.

87. $x + y - 1 = \ln(x^2 + y^2), \quad (1, 0)$

$$x + y - 1 = \ln\left(x^2 + y^2\right), \quad (1, 0)$$

$$1 + y' = \frac{2x + 2yy'}{x^2 + y^2}$$

$$x^2 + y^2 + \left(x^2 + y^2\right)y' = 2x + 2yy'$$

At $(1, 0)$: $1 + y' = 2$

$y' = 1$

Tangent line: $y = x - 1$

88. $y^2 + \ln xy = 2, \quad (e, 1)$

Find the Derivative (5.4)

39. $f(x) = e^{2x}$

$f(x) = e^{2x}$

$f'(x) = 2e^{2x}$

41. $y = e^{\sqrt{x}}$

$y = e^{\sqrt{x}}$

$\frac{dy}{dx} = \frac{e^{\sqrt{x}}}{2\sqrt{x}}$

45. $y = e^x \ln x$

$y = e^x \ln x$

$y' = e^x\left(\frac{1}{x}\right) + e^x \ln x = e^x\left(\frac{1}{x} + \ln x\right)$

49. $g(t) = (e^{-t} + e^t)^3$

$g(t) = \left(e^{-t} + e^t\right)^3$

$g'(t) = 3\left(e^{-t} + e^t\right)^2\left(e^t - e^{-t}\right)$

55. $y = \frac{e^x + 1}{e^x - 1}$

$y = \frac{e^x + 1}{e^x - 1}$

$y' = \frac{\left(e^x - 1\right)e^x - \left(e^x + 1\right)e^x}{\left(e^x - 1\right)^2} = \frac{-2e^x}{\left(e^x - 1\right)^2}$

Find the second derivative

73. $f(x) = (3 + 2x)e^{-3x}$

$f(x) = (3 + 2x)e^{-3x}$

$f'(x) = (3 + 2x)\left(-3e^{-3x}\right) + 2e^{-3x} = (-7 - 6x)e^{-3x}$

$f''(x) = (-7 - 6x)\left(-3e^{-3x}\right) - 6e^{-3x} = 3(6x + 5)e^{-3x}$

Other Bases! Find these derivatives using the rules for bases other than *e*. (5.5)

41. $f(x) = 4^x$

$f(x) = 4^x$

$f'(x) = (\ln 4)4^x$

43. $y = 5^{-4x}$

$y = 5^{-4x}$

$y' = -4(\ln 5)5^{-4x}$

$= \frac{-4 \ln 5}{625^x}$

51. $y = \log_4(5x + 1)$

$y = \log_4(5x + 1)$

$y' = \frac{1}{(5x + 1)\ln 4}(5)$

$= \frac{5}{\ln 4(5x + 1)}$

53. $h(t) = \log_5(4 - t)^2$

$h(t) = \log_5(4 - t)^2 = 2\log_5(4 - t)$

$h'(t) = 2\frac{-1}{\ln(5)(4 - t)} = \frac{2}{(t - 4)\ln 5}$

55. $y = \log_5 \sqrt{x^2 - 1}$

$y = \log_5\sqrt{x^2 - 1} = \frac{1}{2}\log_5\left(x^2 - 1\right)$

$\frac{dy}{dx} = \frac{1}{2} \cdot \frac{2x}{\left(x^2 - 1\right)\ln 5} = \frac{x}{\left(x^2 - 1\right)\ln 5}$

57. $f(x) = \log_2 \frac{x^2}{x - 1}$

$f(x) = \log_2 \frac{x^2}{x - 1} = 2\log_2 x - \log_2(x - 1)$

$f'(x) = \frac{2}{x \ln 2} - \frac{1}{(x - 1)\ln 2} = \frac{x - 2}{(\ln 2)x(x - 1)}$

Use logarithmic differentiation to solve these (i.e. take the natural log first)

67. $y = x^{2/x}$

$$y = x^{2/x}$$

$$\ln y = \frac{2}{x}\ln x$$

$$\frac{1}{y}\left(\frac{dy}{dx}\right) = \frac{2}{x}\left(\frac{1}{x}\right) + \ln x\left(-\frac{2}{x^2}\right) = \frac{2}{x^2}(1 - \ln x)$$

$$\frac{dy}{dx} = \frac{2y}{x^2}(1 - \ln x) = 2x^{(2/x)-2}(1 - \ln x)$$

69. $y = (x - 2)^{x+1}$

$$y = (x - 2)^{x+1}$$

$$\ln y = (x + 1)\ln(x - 2)$$

$$\frac{1}{y}\left(\frac{dy}{dx}\right) = (x + 1)\left(\frac{1}{x - 2}\right) + \ln(x - 2)$$

$$\frac{dy}{dx} = y\left[\frac{x + 1}{x - 2} + \ln(x - 2)\right]$$

$$= (x - 2)^{x+1}\left[\frac{x + 1}{x - 2} + \ln(x - 2)\right]$$