Name: Date:

2G: Derivatives of Inverse Trig. Functions

We have reached the end of our Derivative study!

Our last group of derivatives is inverse Trigonometric functions. We need to think about these because we will need them when we study Integrals later.

A Little Trig. Review

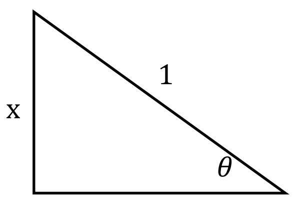
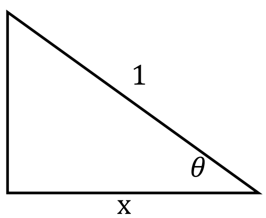
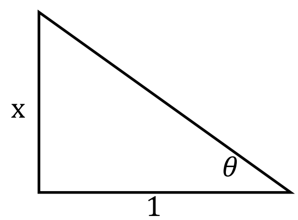
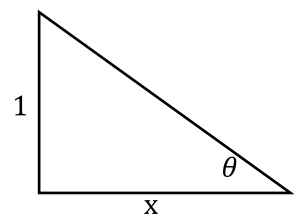
Remember that the inverse trigonometric functions can be written in several forms:

$$\theta = \sin^{-1} x\,, \qquad \theta = \arcsin x\,, \qquad \theta = \operatorname{asin} x$$

In terms of angles, these all mean “Theta is the angle that has a sine of x.”

Try this: In each right triangle below, two sides of the triangle and one angle are labeled.

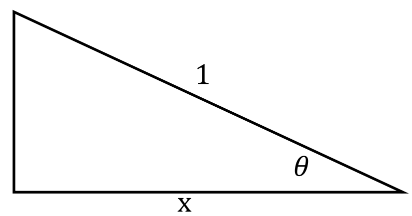
a) Use the labeled sides to write a true equation involving x, θ, and an inverse trig. Function.

 $\cos(\arcsin x)$	 $\sin(\arccos x)$	 $\sec(\arctan x)$	 $\csc(\operatorname{arccot} x)$

We will also need these ones:

$\sec^2(\arctan x) =$

$\csc^2(\operatorname{arccot} x) =$



Derivatives of Inverses

It turns out that there is an important connection between the derivatives of a function and it's inverse.

Derivative of Inverse Theorem

If $g(x) = f^{-1}(x)$ is the inverse of $f(x)$

$$g'(x) = \frac{1}{f'\big(g(x)\big)}$$

Example An increasing function f satisfies $f(10) = 5$ and $f'(10) = 8$.

Find $\frac{d}{dx} f^{-1}(5)$

Now, use this theorem to find the derivative of $\mathbf{arcsin}(x)$ and $\mathbf{arccos}(x)$.

$\frac{d}{dx}[\arcsin(x)] =$

$\frac{d}{dx}[\arccos(x)] =$

We can use the same method to find the derivative of all the inverse trig functions. Here is what we get…

Specific Rule 7: Derivative of Inverse Trigonometric Function.

Let u be a differentiable function of x.

$$\frac{d}{dx}[\arcsin x] = \frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}[\arccos u] = \frac{-1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}[\arctan u] = \frac{1}{1+x^2} \qquad \frac{d}{dx}[\operatorname{arccot} u] = \frac{-1}{1+x^2}$$

$$\frac{d}{dx}[\operatorname{arcsec} u] = \frac{1}{|x|\sqrt{x^2-1}} \qquad \frac{d}{dx}[\operatorname{arccsc} u] = \frac{-1}{|x|\sqrt{x^2-1}}$$

Specific Rule 7: Derivative of Inverse Trigonometric Function.

Let u be a differentiable function of x.

$$\frac{d}{dx}[\arcsin u] = \frac{u'}{\sqrt{1-u^2}} \qquad \frac{d}{dx}[\arccos u] = \frac{-u'}{\sqrt{1-u^2}}$$

$$\frac{d}{dx}[\arctan u] = \frac{u'}{1+u^2} \qquad \frac{d}{dx}[\operatorname{arccot} u] = \frac{-u'}{1+u^2}$$

$$\frac{d}{dx}[\operatorname{arcsec} u] = \frac{u'}{|u|\sqrt{u^2-1}} \qquad \frac{d}{dx}[\operatorname{arccsc} u] = \frac{-u'}{|u|\sqrt{u^2-1}}$$

'Yes, you need to memorize these… but hopefully knowing where they come from (the Pythagorean theorem and the Derivative of Inverses theorem) will help a little. Time to make some notecards!

Try a few: Find these derivatives.

$\frac{d}{dx}[\arcsin(3x^5+4x)]$ $\qquad$ $\frac{d}{dx}[\operatorname{arccot}(e^x)]$ $\qquad$ $\frac{d}{dx}[\operatorname{arcsec}(\ln(2x))]$