

2.5: Proving Statements about segments and angles

How do we **PROVE** that something is true?



Core Concept

Algebraic Properties of Equality

Let a , b , and c be real numbers.

Addition Property of Equality	If $a = b$, then $a + c = b + c$.
Subtraction Property of Equality	If $a = b$, then $a - c = b - c$.
Multiplication Property of Equality	If $a = b$, then $a \cdot c = b \cdot c$, $c \neq 0$.
Division Property of Equality	If $a = b$, then $\frac{a}{c} = \frac{b}{c}$, $c \neq 0$.
Substitution Property of Equality	If $a = b$, then a can be substituted for b (or b for a) in any equation or expressio

Work with a partner. In previous courses, you studied different properties, such as the properties of equality and the Distributive, Commutative, and Associative Properties. Write the property that justifies each of the following solution steps.

Algebraic Step	Justification
$2(x + 3) - 5 = 5x + 4$	Write given equation.
$2x + 6 - 5 = 5x + 4$	<div style="background-color: #f0e68c; height: 15px; width: 100%;"></div>
$2x + 1 = 5x + 4$	<div style="background-color: #f0e68c; height: 15px; width: 100%;"></div>
$2x - 2x + 1 = 5x - 2x + 4$	<div style="background-color: #f0e68c; height: 15px; width: 100%;"></div>
$1 = 3x + 4$	<div style="background-color: #f0e68c; height: 15px; width: 100%;"></div>
$1 - 4 = 3x + 4 - 4$	<div style="background-color: #f0e68c; height: 15px; width: 100%;"></div>
$-3 = 3x$	<div style="background-color: #f0e68c; height: 15px; width: 100%;"></div>
$\frac{-3}{3} = \frac{3x}{3}$	<div style="background-color: #f0e68c; height: 15px; width: 100%;"></div>
$-1 = x$	<div style="background-color: #f0e68c; height: 15px; width: 100%;"></div>
$x = -1$	<div style="background-color: #f0e68c; height: 15px; width: 100%;"></div>

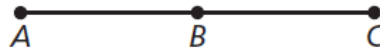
Essential Question

How can you prove a mathematical statement?

Work with a partner. Four steps of a proof are shown. Write the reasons for each statement.

Given $AC = AB + AB$

Prove $AB = BC$

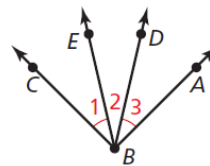


STATEMENTS	REASONS
1. $AC = AB + AB$	1. Given
2. $AB + BC = AC$	2.
3. $AB + AB = AB + BC$	3.
4. $AB = BC$	4.

Work with a partner. Six steps of a proof are shown. Complete the statements that correspond to each reason

Given $m\angle 1 = m\angle 3$

Prove $m\angle EBA = m\angle CBD$

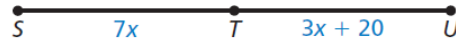


STATEMENTS	REASONS
1. 	1. Given
2. $m\angle EBA = m\angle 2 + m\angle 3$	2. Angle Addition Postulate (Post.1.4)
3. $m\angle EBA = m\angle 2 + m\angle 1$	3. Substitution Property of Equality
4. $m\angle EBA =$ 	4. Commutative Property of Addition
5. $m\angle 1 + m\angle 2 =$ 	5. Angle Addition Postulate (Post.1.4)
6. 	6. Transitive Property of Equality

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Six steps of a two-column proof are shown. Copy and complete the proof.

Given T is the midpoint of \overline{SU} .



Prove $x = 5$

STATEMENTS	REASONS
1. T is the midpoint of \overline{SU} .	1. _____
2. $\overline{ST} \cong \overline{TU}$	2. Definition of midpoint
3. $ST = TU$	3. Definition of congruent segments
4. $7x = 3x + 20$	4. _____
5. _____	5. Subtraction Property of Equality
6. $x = 5$	6. _____

Concept Summary

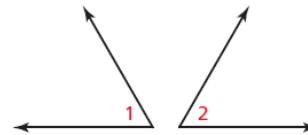
Writing a Two-Column Proof

In a proof, you make one statement at a time until you reach the conclusion. Because you make statements based on facts, you are using deductive reasoning. Usually the first statement-and-reason pair you write is given information.

Proof of the Symmetric Property of Angle Congruence

Given $\angle 1 \cong \angle 2$

Prove $\angle 2 \cong \angle 1$



Copy or draw diagrams and label given information to help develop proofs. Do not mark or label the information in the Prove statement on the diagram.

STATEMENTS	REASONS
1. $\angle 1 \cong \angle 2$	1. Given
2. $m\angle 1 = m\angle 2$	2. Definition of congruent angles
3. $m\angle 2 = m\angle 1$	3. Symmetric Property of Equality
4. $\angle 2 \cong \angle 1$	4. Definition of congruent angles

statements based on facts that you know or on conclusions from deductive reasoning

The number of statements will vary.

Remember to give a reason for the last statement.

definitions, postulates, or proven theorems that allow you to state the corresponding statement

Theorems

Theorem 2.1 Properties of Segment Congruence

Segment congruence is reflexive, symmetric, and transitive.

Reflexive For any segment AB , $\overline{AB} \cong \overline{AB}$.

Symmetric If $\overline{AB} \cong \overline{CD}$, then $\overline{CD} \cong \overline{AB}$.

Transitive If $\overline{AB} \cong \overline{CD}$ and $\overline{CD} \cong \overline{EF}$, then $\overline{AB} \cong \overline{EF}$.

Proofs Ex. 11, p. 103; Example 3, p. 101; Chapter Review 2.5 Example, p. 118

Theorem 2.2 Properties of Angle Congruence

Angle congruence is reflexive, symmetric, and transitive.

Reflexive For any angle A , $\angle A \cong \angle A$.

Symmetric If $\angle A \cong \angle B$, then $\angle B \cong \angle A$.

Transitive If $\angle A \cong \angle B$ and $\angle B \cong \angle C$, then $\angle A \cong \angle C$.

Proofs Ex. 25, p. 118; 2.5 Concept Summary, p. 102; Ex. 12, p. 103

Name the property that the statement illustrates.

a. If $\angle T \cong \angle V$ and $\angle V \cong \angle R$, then $\angle T \cong \angle R$.

b. If $\overline{JL} \cong \overline{YZ}$, then $\overline{YZ} \cong \overline{JL}$.

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Write a two-column proof for the Symmetric Property of Segment Congruence.

Given $\overline{LM} \cong \overline{NP}$

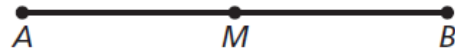
Prove $\overline{NP} \cong \overline{LM}$



Prove this property of midpoints: If you know that M is the midpoint of \overline{AB} , prove that AB is two times AM and AM is one-half AB .

Given M is the midpoint of \overline{AB} .

Prove $AB = 2AM$, $AM = \frac{1}{2}AB$



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Name the property that the statement illustrates.

2. $\overline{GH} \cong \overline{GH}$

3. If $\angle K \cong \angle P$, then $\angle P \cong \angle K$.

4. Look back at Example 4. What would be different if you were proving that $AB = 2 \cdot MB$ and that $MB = \frac{1}{2}AB$ instead?

Prove the Reflexive Property of Segment Congruence. (See Exercise 11.)