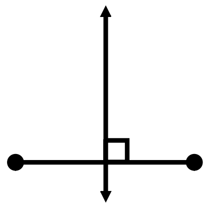
6.1: Perpendicular and Angle Bisectors

Geometry

Essential Question

What conjectures can you make about a point on the perpendicular bisector of a segment and a point on the bisector of an angle?

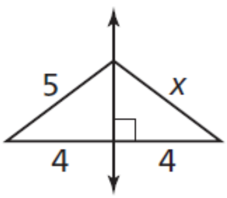


Essential Question

Warmup

The diagram includes a pair of congruent triangles. Use the congruent triangles to find the value of *x* in the diagram.

1.



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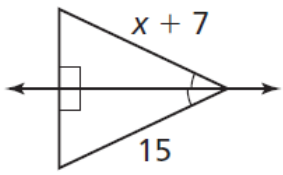

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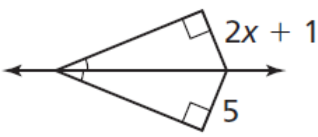

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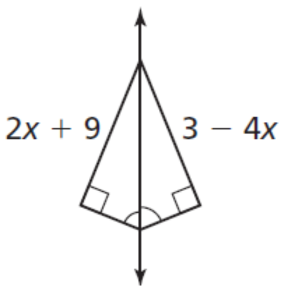
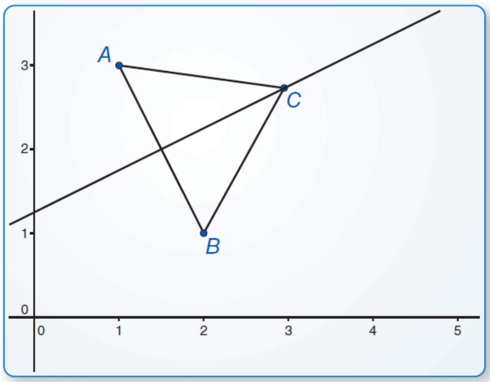


Warm Up

Exploration 1: Perpendicular Bisectors

Work with a partner. Use dynamic geometry software.

a. Draw any segment and label it $\overline{AB}$. Construct the perpendicular bisector of $\overline{AB}$.



Sample
Points
$A(1, 3)$
$B(2, 1)$
$C(2.95, 2.73)$
Segments
$AB = 2.24$
$CA = ?$
$CB = ?$
Line
$-x + 2y = 2.5$

b. Label a point C that is on the perpendicular bisector of $\overline{AB}$ but is not on $\overline{AB}$.

Exploration 1

c. Draw $\overline{CA}$ and $\overline{CB}$ and find their lengths. Then move point C to other locations on the perpendicular bisector and note the lengths of $\overline{CA}$ and $\overline{CB}$.

d. Repeat parts (a)–(c) with other segments. Describe any relationship(s) you notice.

Exploration 1c-d

Exploration 2: Angle Bisectors

Work with a partner. Use dynamic geometry software.

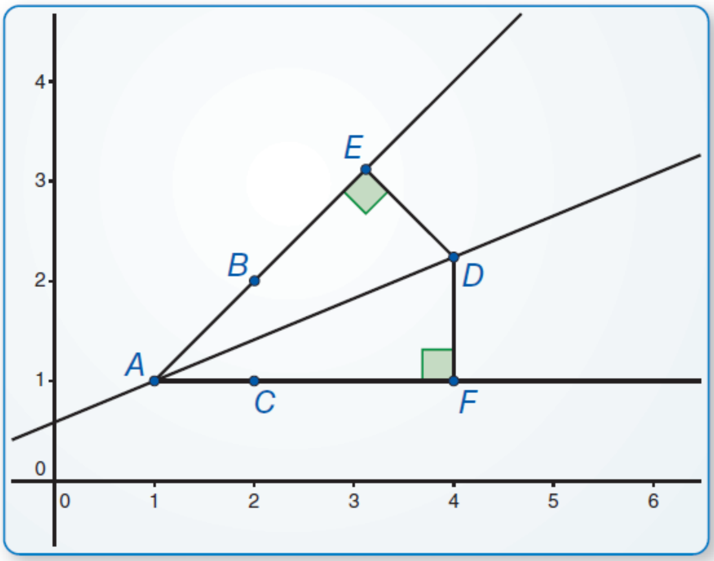
a. Draw two rays $\overrightarrow{AB}$ and $\overrightarrow{AC}$ to form $\angle BAC$. Construct the bisector of $\angle BAC$.

b. Label a point *D* on the bisector of $\angle BAC$.

c. Construct and find the lengths of the perpendicular segments from *D* to the sides of $\angle BAC$. Move point *D* along the angle bisector and note how the lengths change.

d. Repeat parts (a)–(c) with other angles. Describe any relationship(s) you notice.

Exploration 2



Sample

Points

$A(1, 1)$
$B(2, 2)$
$C(2, 1)$
$D(4, 2.24)$

Rays

$AB = -x + y = 0$
$AC = y = 1$

Line

$-0.38x + 0.92y = 0.54$

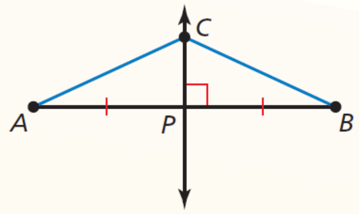
Exploration 2 Graph

Theorems

Theorem 6.1 Perpendicular Bisector Theorem

In a plane, if a point lies on the perpendicular bisector of a segment, then it is equidistant from the endpoints of the segment.

If $\overleftrightarrow{CP}$ is the $\perp$ bisector of $\overline{AB}$, then $CA = CB$.

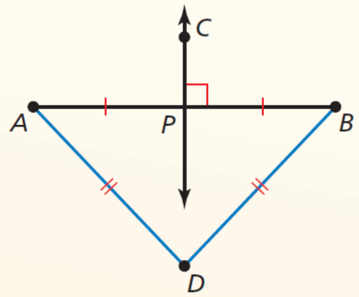


Proof p. 302

Theorem 6.2 Converse of the Perpendicular Bisector Theorem

In a plane, if a point is equidistant from the endpoints of a segment, then it lies on the perpendicular bisector of the segment.

If $DA = DB$, then point D lies on the $\perp$ bisector of $\overline{AB}$.

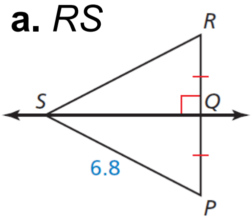


Proof Ex. 32, p. 308

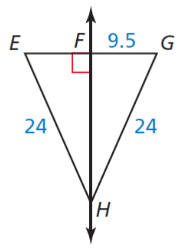
Theorem

Find each measure.

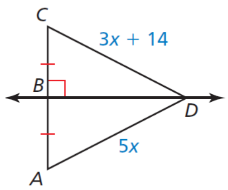
a. *RS*



b. *EG*



c. *AD*



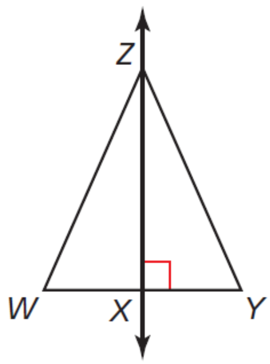
Example 1a-b

Is there enough information in the diagram to conclude that point *N* lies on the perpendicular bisector of $\overline{KM}$?

Example 2

Use the diagram and the given information to find the indicated measure.

1. $\overleftrightarrow{ZX}$ is the perpendicular bisector of $\overline{WY}$, and *YZ* = 13.75. Find *WZ*.



2. $\overleftrightarrow{ZX}$ is the perpendicular bisector of $\overline{WY}$, *WZ* = 4*n* − 13, and *YZ* = *n* + 17. Find *YZ*.

3. Find *WX* when *WZ* = 20.5, *WY* = 14.8, and *YZ* = 20.5.

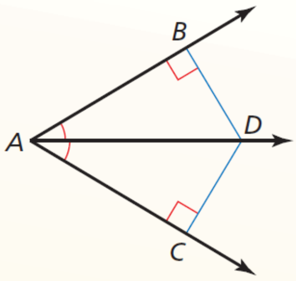
Monitoring Progress 1-3

Theorems

Theorem 6.3 Angle Bisector Theorem

If a point lies on the bisector of an angle, then it is equidistant from the two sides of the angle.

If $\overrightarrow{AD}$ bisects $\angle BAC$ and $\overline{DB} \perp \overrightarrow{AB}$ and $\overline{DC} \perp \overrightarrow{AC}$, then $DB = DC$.

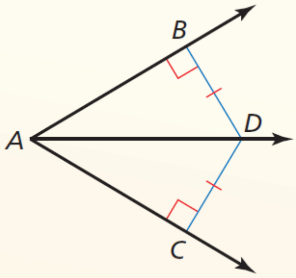


Proof Ex. 33(a), p. 308

Theorem 6.4 Converse of the Angle Bisector Theorem

If a point is in the interior of an angle and is equidistant from the two sides of the angle, then it lies on the bisector of the angle.

If $\overline{DB} \perp \overrightarrow{AB}$ and $\overline{DC} \perp \overrightarrow{AC}$ and $DB = DC$, then $\overrightarrow{AD}$ bisects $\angle BAC$.

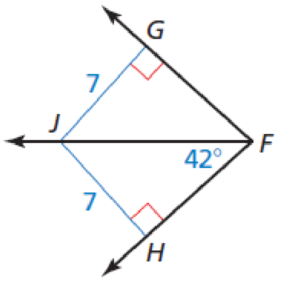


Proof Ex. 33(b), p. 308

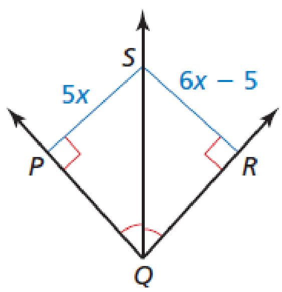
Theorem

Find each measure.

a. $m\angle GFJ$

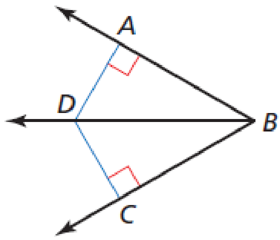


b. RS



Example 3a

Use the diagram and the given information to find the indicated measure.



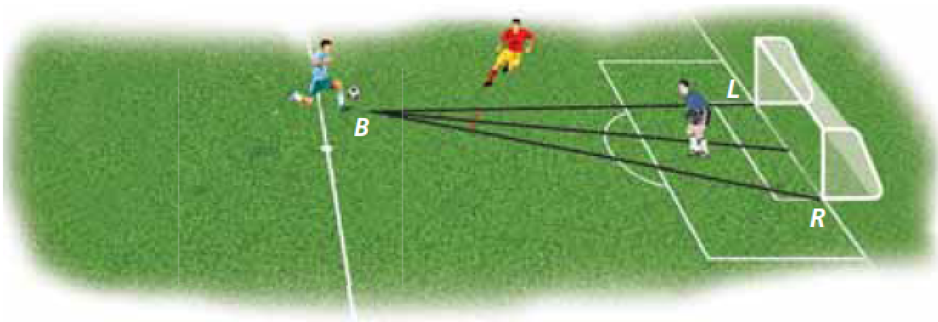
4. $\overrightarrow{BD}$ bisects $\angle ABC$, and $DC = 6.9$. Find DA.

5. $\overrightarrow{BD}$ bisects $\angle ABC$, $AD = 3z + 7$, and $CD = 2z + 11$. Find CD.

6. Find $m\angle ABC$ when $AD = 3.2$, $CD = 3.2$, and $m\angle DBC = 39°$.

Monitoring Progress 4-6

A soccer goalie's position relative to the ball and goalposts forms congruent angles, as shown. Will the goalie have to move farther to block a shot toward the right goalpost R or the left goalpost L?



Example 4