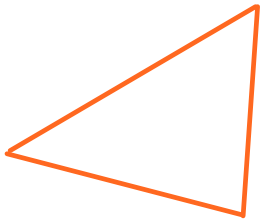
6.2: Bisectors in Triangles

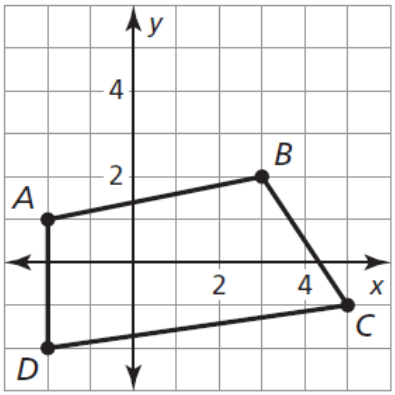
Essential Question

What conjectures can you make about the perpendicular bisectors and the angle bisectors of a triangle?

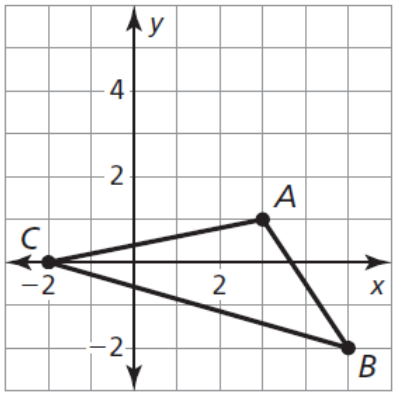


Graph the polygon and its image after a reflection in the given line.

1. y = x

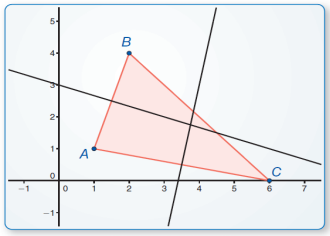


2. y = 2



Work with a partner. Use dynamic geometry software.
Draw any △ABC.

a. Construct the perpendicular bisectors of all three sides of △ABC. Then drag the vertices to change △ABC. What do you notice about the perpendicular bisectors?



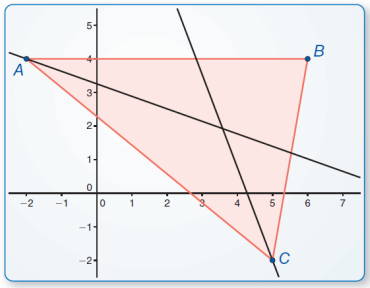
Sample
Points
$A(1, 1)$
$B(2, 4)$
$C(6, 0)$
Segments
$BC = 5.66$
$AC = 5.10$
$AB = 3.16$
Lines
$x + 3y = 9$
$-5x + y = -17$

b. Label a point D at the intersection of the perpendicular bisectors.

c. Draw the circle with center D through vertex A of △ABC. Then drag the vertices to change △ABC. What do you notice?

Work with a partner. Use dynamic geometry software. Draw any △ABC.

a. Construct the angle bisectors of all three angles of △ABC. Then drag the vertices to change △ABC. What do you notice about the angle bisectors?



Sample
Points
$A(-2, 4)$
$B(6, 4)$
$C(5, -2)$
Segments
$BC = 6.08$
$AC = 9.22$
$AB = 8$
Lines
$0.35x + 0.94y = 3.06$
$-0.94x - 0.34y = -4.02$

b. Label a point D at the intersection of the angle bisectors.

c. Find the distance between D and $\overline{AB}$. Draw the circle with center D and this distance as a radius . Then drag the vertices to change △ABC. What do you notice?

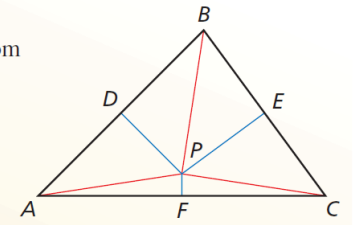
Theorems

Theorem 6.5 Circumcenter Theorem

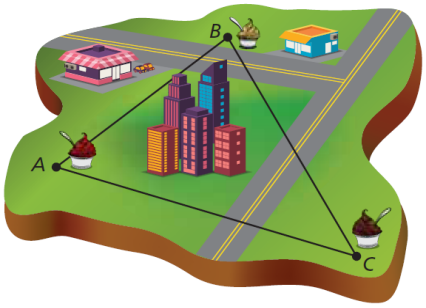
The circumcenter of a triangle is equidistant from the vertices of the triangle.

If $\overline{PD}$, $\overline{PE}$, and $\overline{PF}$ are perpendicular bisectors, then $PA = PB = PC$.

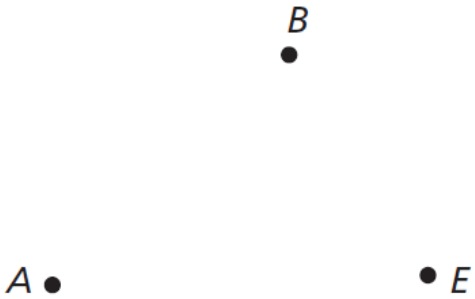
Proof p. 310



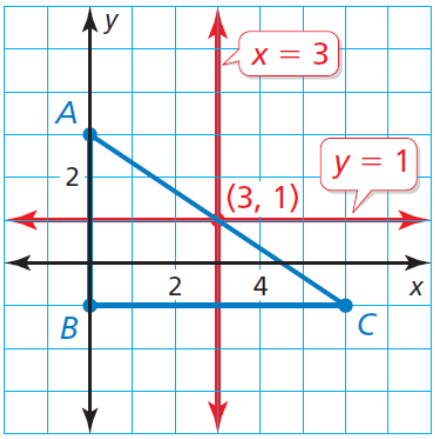
Three snack carts sell frozen yogurtfrom points A, B, and C outside a city. Each of the three carts is the same distance from the frozen yogurt distributor. Find the location of the distributor.



1. Three snack carts sell hot pretzels from points A, B, and E What is the location of the pretzel distributor if it is equidistant from the three carts? Sketch the triangle and show the location.



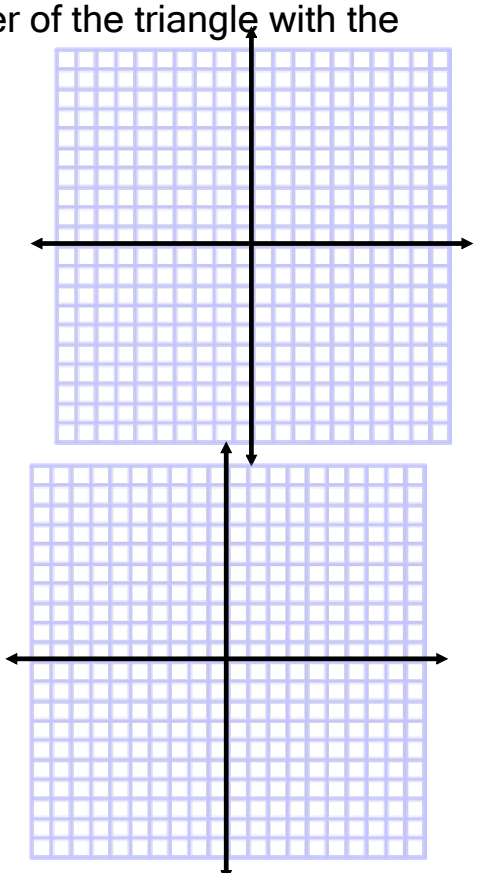
Find the coordinates of the circumcenter of $\triangle ABC$ with vertices $A(0, 3), B(0, -1)$, and $C(6, -1)$.



Find the coordinates of the circumcenter of the triangle with the given vertices.

2. $R(-2, 5), S(-6, 5), T(-2, -1)$

3. $W(-1, 4), X(1, 4), Y(1, -6)$



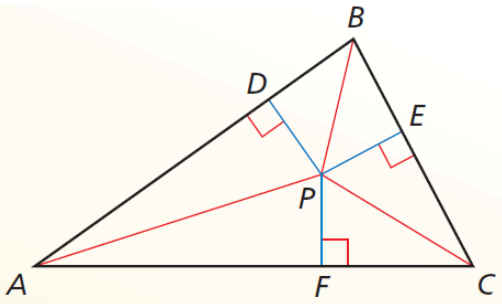
Theorem

Theorem 6.6 Incenter Theorem

The incenter of a triangle is equidistant from the sides of the triangle.

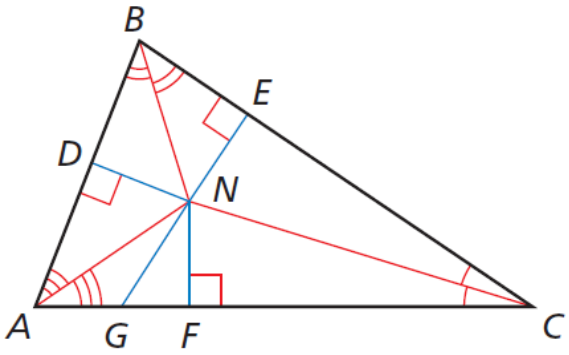
If $\overline{AP}$, $\overline{BP}$, and $\overline{CP}$ are angle bisectors of $\triangle ABC$, then $PD = PE = PF$.

Proof Ex. 38, p. 317



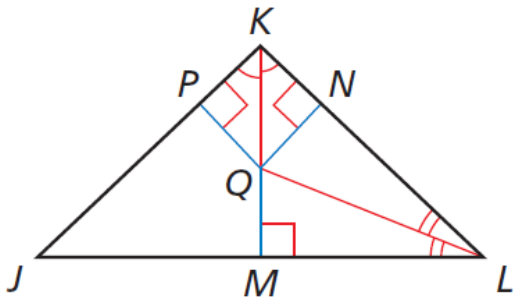
In the figure shown, ND = 5x − 1 and NE = 2x + 11.

a. Find NF.

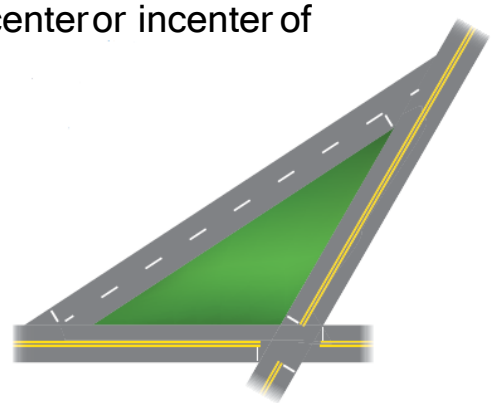


b. Can NG be equal to 18? Explain your reasoning.

4. In the figure shown, QM = 3x + 8 and QN = 7x + 2. Find QP.



A city wants to place a lamppost on the boulevard shown so that the lamppost is the same distance from all three streets. Should the location of the lamppost be at the circumcenter or incenter of the triangular boulevard? Explain.



Draw a sketch to show the location L of the lamppost .