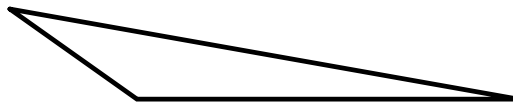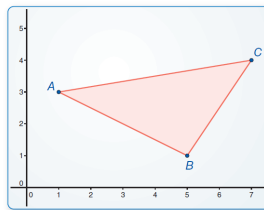# 6.5: Inequalities in Triangles

Essential Question

How are the sides related to the angles of a triangle? How are any two sides of a triangle related to the third side?

Exploration 1

Work with a partner.   Use dynamic geometry software. Draw any scalene △ABC.

a. Find the side lengths and angle measures of the triangle.

Sample

Points	Angles
A(1, 3)	m∠A = ?
B(5, 1)	m∠B = ?
C(7, 4)	m∠C = ?

Segments
BC = ?
AC = ?
AB = ?

b. Order the side lengths. Order the angle measures. What do you observe?

	Smallest Angle	Smallest Side	Largest Angle	Largest Angle
Triangle 1				
Triangle 2				
Triangle 3				

c. Drag the vertices of △ABC to form new triangles. Fill in the table for these new triangles. Write a conjecture about your findings.
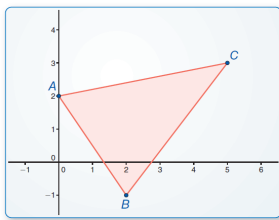
Exploration 2

Work with a partner.   Use dynamic geometry software.
Draw any △ABC.

a. Find the side lengths of the triangle.

b. Compare each side length with the sum of the other
two side lengths.

Side 1+Side 2	Side 3

Sample
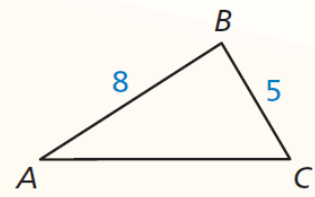Points
$A(0, 2)$
$B(2, -1)$
$C(5, 3)$
Segments
$BC = ?$
$AC = ?$
$AB = ?$

Theorems

Theorem 6.9   Triangle Longer Side Theorem

If one side of a triangle is longer than another side,
then the angle opposite the longer side is larger than
the angle opposite the shorter side.
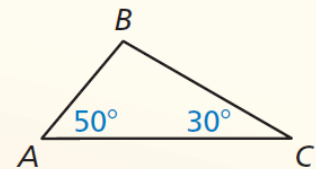
Proof Ex. 43, p. 342

$AB > BC$, so $m\angle C > m\angle A$.

Theorem 6.10   Triangle Larger Angle Theorem

If one angle of a triangle is larger than another angle,
then the side opposite the larger angle is longer than
the side opposite the smaller angle.

Proof p. 337

$m\angle A > m\angle C$, so $BC > AB$.

You are constructing a stage prop that shows a large triangular mountain. The bottom edge of the mountain is about 32 feet long, the left slope is about 24 feet long, and the right slope is about 26 feet long. List the angles of △JKL in order from smallest to largest.
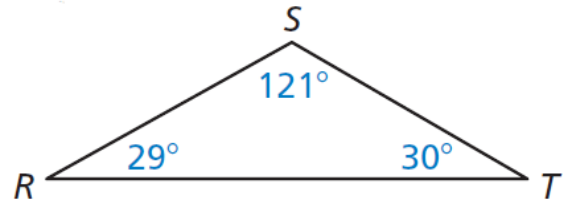

List the sides of △DEF in order from shortest to longest.
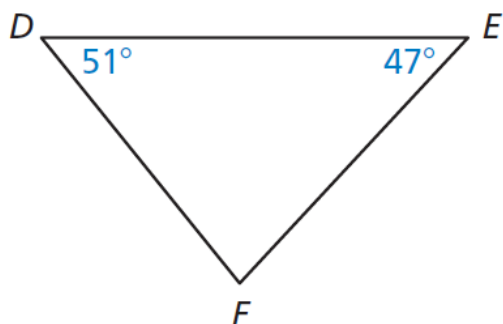
2. List the angles of △PQR in order from smallest to largest.

Q

6.8

P

6.1

5.9

R
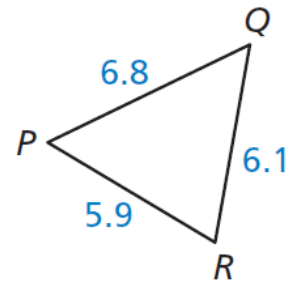
3. List the sides of △RST in order

from shortest to longest.

S

121°

29°   30°

R   T

Theorem

Theorem 6.11   Triangle Inequality Theorem

The sum of the lengths of any two sides of
a triangle is greater than the length of the
third side.

$AB + BC > AC$ $AC + BC > AB$ $AB + AC > BC$

Proof   Ex. 47, p. 342

B

A

C

Decide whether it is possible to construct a triangle with the given side lengths.

Explain your reasoning.

5. 4 ft, 9 ft, 10 ft   6. 8 m, 9 m, 18 m   7. 5 cm, 7 cm, 12 cm

A triangle has one side of length 14 and another side of length 9. Describe the possible lengths of the third side.

4. A triangle has one side of length 12 inches and another side of length 20 inches. Describe the possible lengths of the third side.