



7.3 Proving that a Quadrilateral is a parallelogram

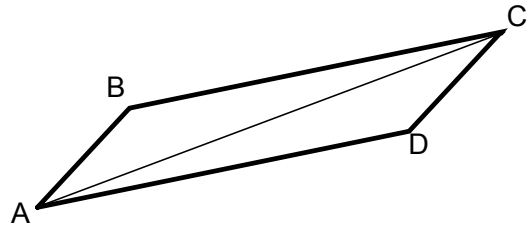
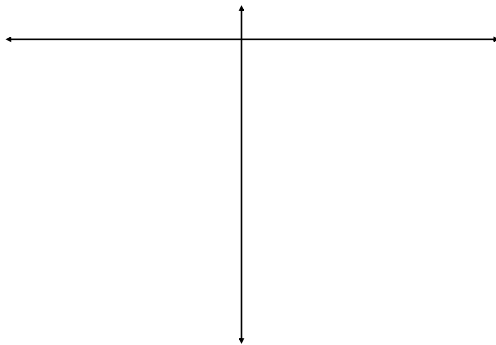
Essential Question

How can you prove that a quadrilateral is a parallelogram?

Warmup:

Given: ABCD is a parallelogram

Prove: $\triangle ABC \cong \triangle CDA$



Exploration 1

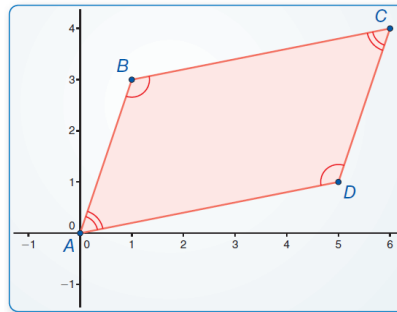
1. Get 2 straws, a string, and scissors
2. Hold the 2 straws together and cut them at the same time to get 2 pairs of congruent pieces
3. String them together alternating lengths.
4. What shape do you get? Will it always be this shape?

State a conjecture:

Work with a partner. Use dynamic geometry software.

a. Construct any quadrilateral $ABCD$ whose opposite angles are congruent.

b. Is the quadrilateral a parallelogram? Justify your answer.



Sample

Points

$A(0, 0)$

$B(1, 3)$

$C(6, 4)$

$D(5, 1)$

Angles

$\angle A = 60.26^\circ$

$\angle B = 119.74^\circ$

$\angle C = 60.26^\circ$

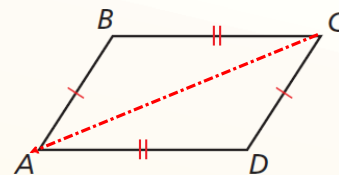
$\angle D = 119.74^\circ$

Theorems

Theorem 7.7 Parallelogram Opposite Sides Converse

If both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram.

If $\overline{AB} \cong \overline{CD}$ and $\overline{BC} \cong \overline{DA}$, then $ABCD$ is a parallelogram.

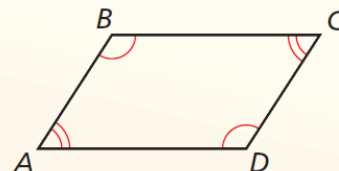


Theorem 7.8 Parallelogram Opposite Angles Converse

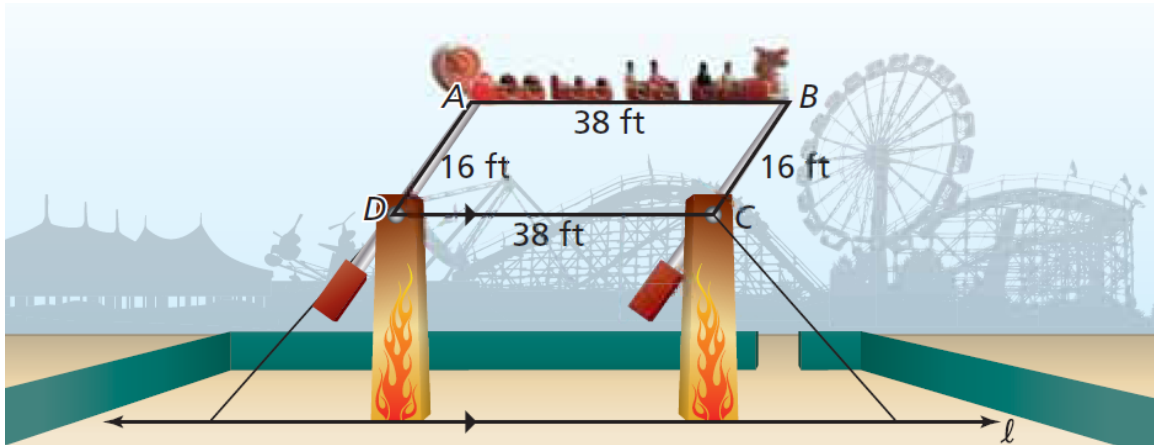
If both pairs of opposite angles of a quadrilateral are congruent, then the quadrilateral is a parallelogram.

If $\angle A \cong \angle C$ and $\angle B \cong \angle D$, then $ABCD$ is a parallelogram.

Proof Ex. 39, p. 383



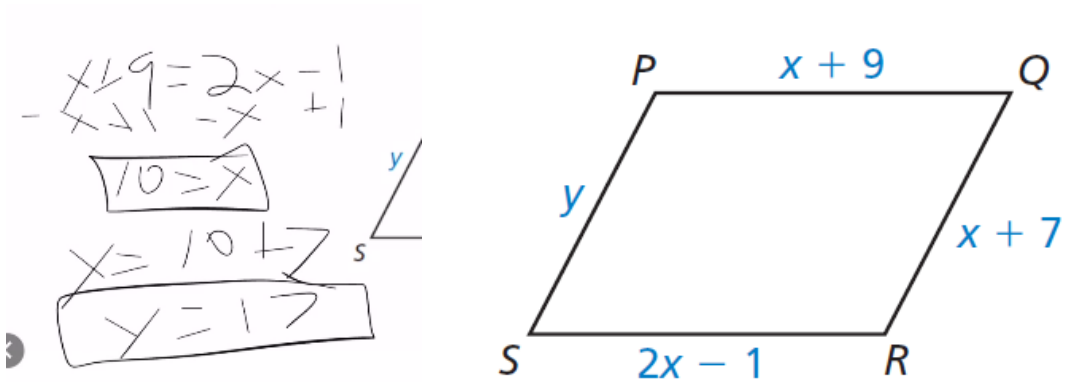
An amusement park ride has a moving platform attached to four swinging arms. The platform swings back and forth, higher and higher, until it goes over the top and around in a circular motion. In the diagram below, \overline{AD} and \overline{BC} represent two of the swinging arms, and \overline{DC} is parallel to the ground (line ℓ). Explain why the moving platform \overline{AB} is always parallel to the ground.



1. In quadrilateral $WXYZ$, $m\angle W = 42^\circ$, $m\angle X = 138^\circ$, and $m\angle Y = 42^\circ$. Find $m\angle Z$.

Is $WXYZ$ a parallelogram? Explain your reasoning.

For what values of x and y is quadrilateral $PQRS$ a parallelogram?



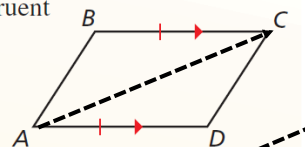
Theorems

Theorem 7.9 Opposite Sides Parallel and Congruent Theorem

If one pair of opposite sides of a quadrilateral are congruent and parallel, then the quadrilateral is a parallelogram.

If $\overline{BC} \parallel \overline{AD}$ and $\overline{BC} \cong \overline{AD}$, then $ABCD$ is a parallelogram.

Proof Ex. 40, p. 383

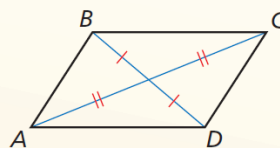


Theorem 7.10 Parallelogram Diagonals Converse

If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.

If \overline{BD} and \overline{AC} bisect each other, then $ABCD$ is a parallelogram.

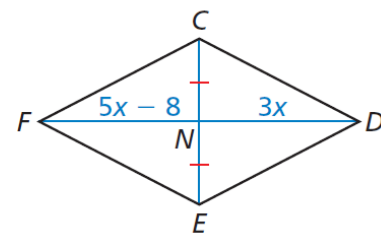
Proof Ex. 41, p. 383



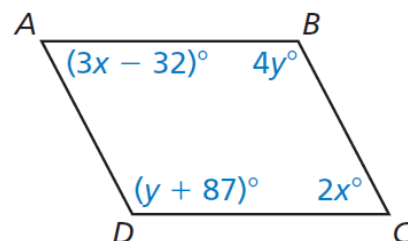
The doorway shown is part of a building in England. Over time, the building has leaned sideways. Explain how you know that $SV = TU$.



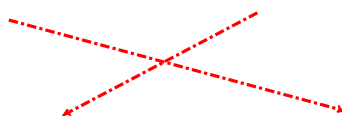
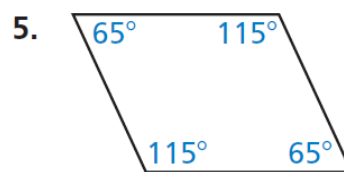
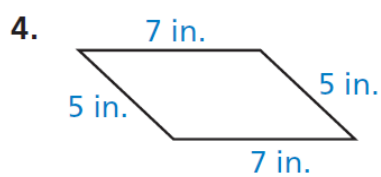
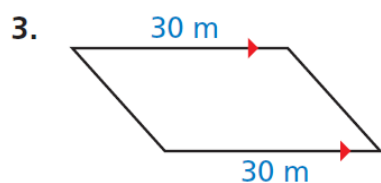
For what value of x is quadrilateral $CDEF$ a parallelogram?



For what values of x and y is quadrilateral $ABCD$ a parallelogram? Explain your reasoning.



State the theorem you can use to show that the quadrilateral is a parallelogram.



Show that quadrilateral $ABCD$ is a parallelogram.

