

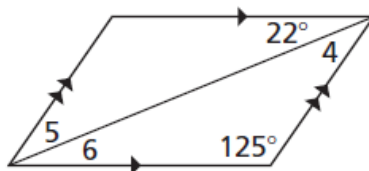
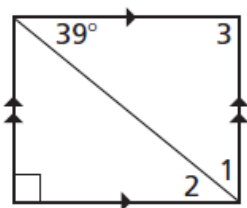


Essential Question

What are the properties of the diagonals of rectangles, rhombuses, and squares?

Warmup

Use the diagrams to determine the measure of each angle.



1. $m\angle 1$

2. $m\angle 2$

3. $m\angle 3$

4. $m\angle 4$

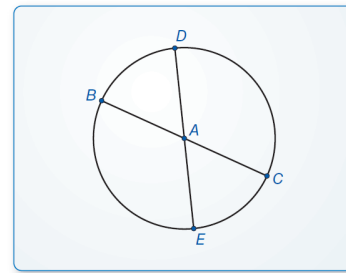
5. $m\angle 5$

6. $m\angle 6$

Work with a partner. Use dynamic geometry software.

a. Draw a circle with center A.

Sample



b. Draw two diameters of the circle.

Label the endpoints B, C, D, and E

c. Draw quadrilateral BDCE

d. Is BDCEa parallelogram? rectangle? rhombus? square?

Explain your reasoning.

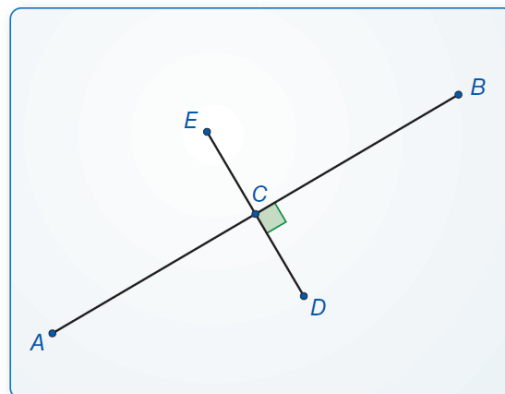
e. Repeat parts (a)-(d) for several other circles. Write a conjecture based on your results.

Exploration 2

Work with a partner. Use dynamic geometry software.

a. Construct two segments that are perpendicular bisectors of each other. Label the endpoints A, B, D, and E Label the intersection C.

Sample



b. Draw quadrilateral AEBC

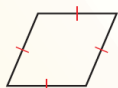
c. Is AEBCa parallelogram? rectangle? rhombus? square?

Explain your reasoning

d. Repeat parts (a)-(c) for several other segments. Write a conjecture based on your results.

Core Concept

Rhombuses, Rectangles, and Squares



A **rhombus** is a parallelogram with four congruent sides.



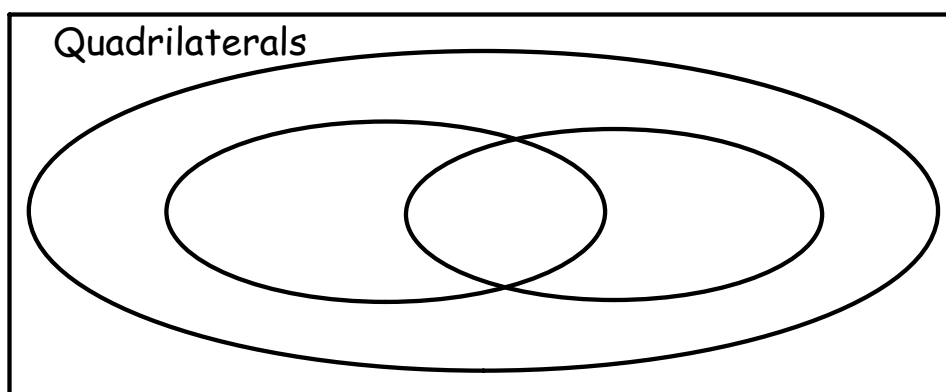
A **rectangle** is a parallelogram with four right angles.



A **square** is a parallelogram with four congruent sides and four right angles.

Place these names in the Venn Diagram:

Parallelograms Rectangles Rhombus' Squares



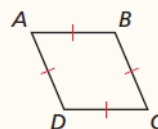
Corollaries

Corollary 7.2 Rhombus Corollary

A quadrilateral is a rhombus if and only if it has four congruent sides.

\overline{ABCD} is a rhombus if and only if $\overline{AB} \cong \overline{BC} \cong \overline{CD} \cong \overline{AD}$.

Proof Ex. 81, p. 396

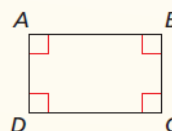


Corollary 7.3 Rectangle Corollary

A quadrilateral is a rectangle if and only if it has four right angles.

$ABCD$ is a rectangle if and only if $\angle A$, $\angle B$, $\angle C$, and $\angle D$ are right angles.

Proof Ex. 82, p. 396

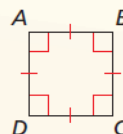


Corollary 7.4 Square Corollary

A quadrilateral is a square if and only if it is a rhombus and a rectangle.

$ABCD$ is a square if and only if $\overline{AB} \cong \overline{BC} \cong \overline{CD} \cong \overline{AD}$ and $\angle A$, $\angle B$, $\angle C$, and $\angle D$ are right angles.

Proof Ex. 83, p. 396



For any rhombus $QRST$ decide whether the statement is always or sometimes true.

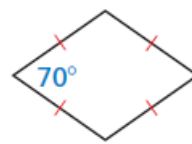
Draw a diagram and explain your reasoning.

a. $\angle Q \cong \angle S$

b. $\angle Q \cong \angle R$

Classify the special quadrilateral.

Explain your reasoning.



1. For any square $JKLM$ is it always or sometimes true that $\overline{JK} \perp \overline{LM}$? Explain your reasoning.

2. For any rectangle $EFGH$ is it always or sometimes true that $\overline{FG} \cong \overline{GH}$? Explain your reasoning.

3. A quadrilateral has four congruent sides and four congruent angles. Sketch the quadrilateral and classify it.

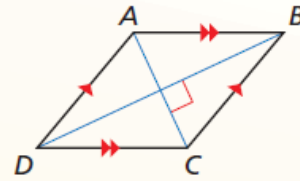
Theorems

Theorem 7.11 Rhombus Diagonals Theorem

A parallelogram is a rhombus if and only if its diagonals are perpendicular.

$\square ABCD$ is a rhombus if and only if $\overline{AC} \perp \overline{BD}$.

Proof p. 390; Ex. 72, p. 395

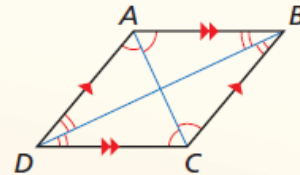


Theorem 7.12 Rhombus Opposite Angles Theorem

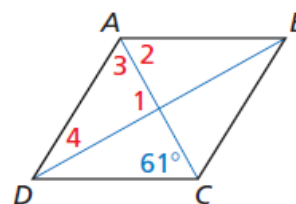
A parallelogram is a rhombus if and only if each diagonal bisects a pair of opposite angles.

$\square ABCD$ is a rhombus if and only if \overline{AC} bisects $\angle BCD$ and $\angle BAD$, and \overline{BD} bisects $\angle ABC$ and $\angle ADC$.

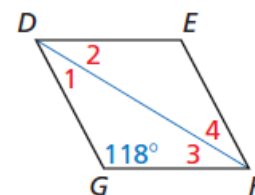
Proof Exs. 73 and 74, p. 395



Find the measures of the numbered angles in rhombus ABCD.



4. In Example 3, what is $m\angle ADC$ and $m\angle BCD$?



5. Find the measures of the numbered angles in rhombus DEFG

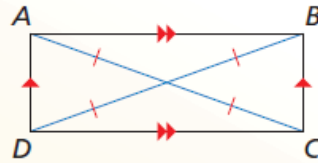
Theorem

Theorem 7.13 Rectangle Diagonals Theorem

A parallelogram is a rectangle if and only if its diagonals are congruent.

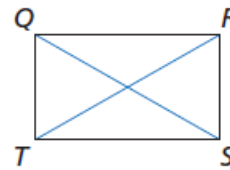
$\square ABCD$ is a rectangle if and only if $\overline{AC} \cong \overline{BD}$.

Proof Exs. 87 and 88, p. 396



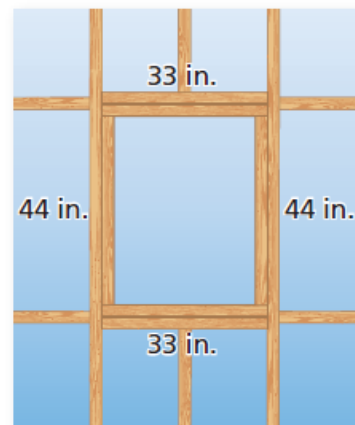
In rectangle QRST, $QS = 5x - 31$ and $RT = 2x + 11$.

Find the lengths of the diagonals of QRST



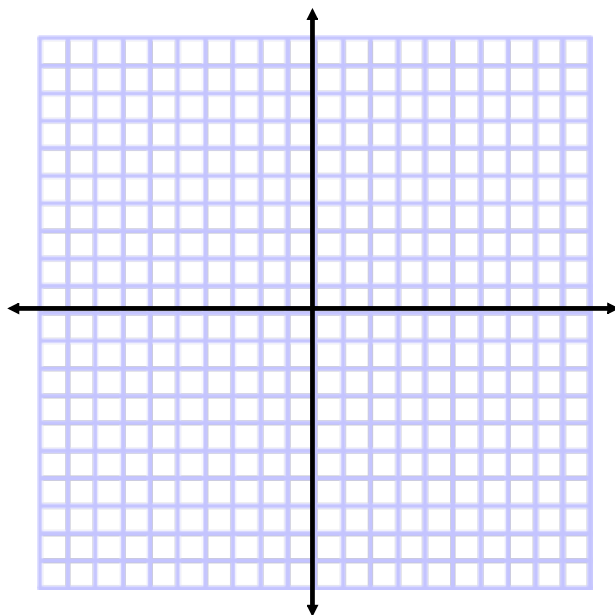
You are building a frame for a window. The window will be installed in the opening shown in the diagram.

a. The opening must be a rectangle. Given the measurements in the diagram, can you assume that it is? Explain.



b. You measure the diagonals of the opening. The diagonals are 54.8 inches and 55.3 inches. What can you conclude about the shape of the opening?

Decide whether $\square ABCD$ with vertices $A(-2, 6)$, $B(6, 8)$, $C(4, 0)$, and $D(-4, -2)$ is a rectangle, a rhombus, or a square. Give all names that apply.



8. Decide whether $\square PQRS$ with vertices $P(-5, 2)$, $Q(0, 4)$, $R(2, -1)$, and $S(-3, -3)$ is a rectangle, a rhombus, or a square. Give all names that apply.

