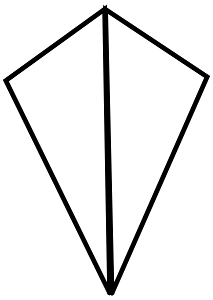
7.5 Trapezoids and Kites

Essential Question

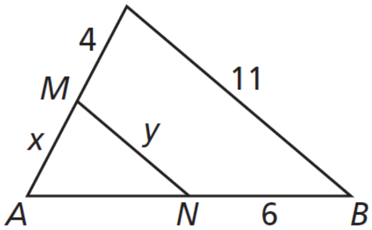
What are some properties of trapezoids and kites?



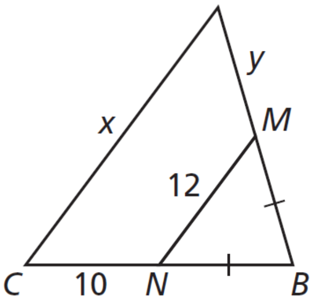
Warmup

$\overline{MN}$ is a midsegment of $\triangle ABC$. Find the values of x and y.

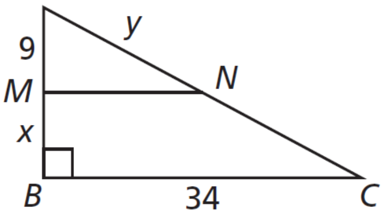
1.



2.



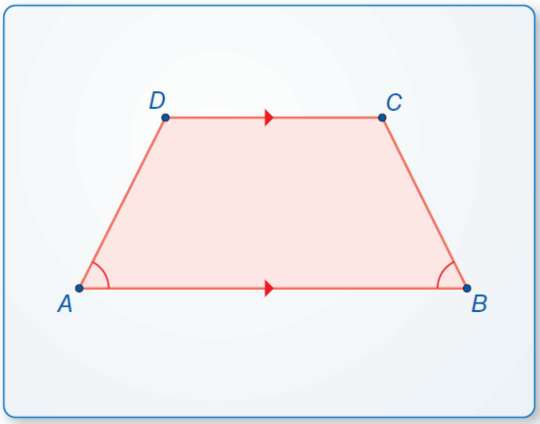
3.



Work with a partner. Use dynamic geometry software.

a. Construct a trapezoid whose base angles are congruent. Explain your process.

Sample

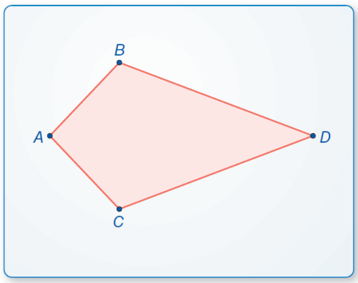


b. Is the trapezoid isosceles? Justify your answer.

Work with a partner. Use dynamic geometry software.

a. Construct a kite.

Sample



b. Measure the angles of the kite. What do you observe?

c. Repeat parts (a) and (b) for several other trapezoids. Write a conjecture based on your results.

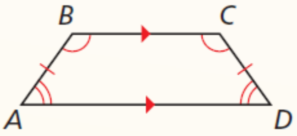
Theorems

Theorem 7.14 Isosceles Trapezoid Base Angles Theorem

If a trapezoid is isosceles, then each pair of base angles is congruent.

If trapezoid *ABCD* is isosceles, then $\angle A \cong \angle D$ and $\angle B \cong \angle C$.

Proof Ex. 39, p. 405



Theorem 7.15 Isosceles Trapezoid Base Angles Converse

If a trapezoid has a pair of congruent base angles, then it is an isosceles trapezoid.

If $\angle A \cong \angle D$ (or if $\angle B \cong \angle C$), then trapezoid *ABCD* is isosceles.

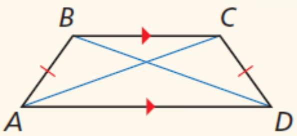
Proof Ex. 40, p. 405

Theorem 7.16 Isosceles Trapezoid Diagonals Theorem

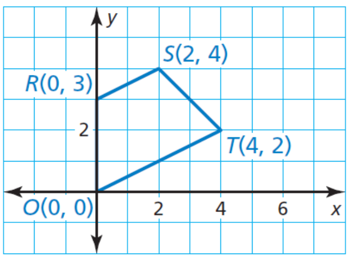
A trapezoid is isosceles if and only if its diagonals are congruent.

Trapezoid *ABCD* is isosceles if and only if $\overline{AC} \cong \overline{BD}$.

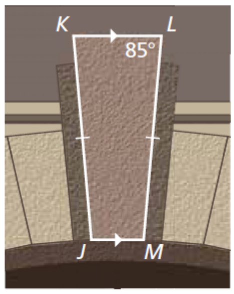
Proof Ex. 51, p. 406



Show that ORSTis a trapezoid. Then decide whether it is isosceles.



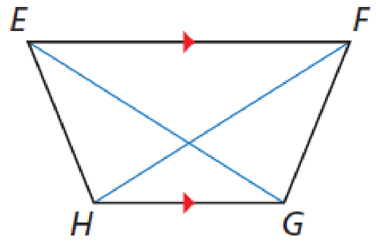
The stone above the arch in the diagram is an isosceles trapezoid. Find m$\angle$K, m$\angle$M, and m$\angle$J.



In Exercises 2 and 3, use trapezoid EFGH

2. If EFGH is an isosceles trapezoid is EG = FH?

Explain.



3. If m∠HEF = 70° and m∠FGH = 110°,

is trapezoid EFGH isosceles? Explain.

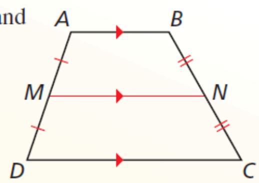
Theorem

Theorem 7.17 Trapezoid Midsegment Theorem

The midsegment of a trapezoid is parallel to each base, and its length is one-half the sum of the lengths of the bases.

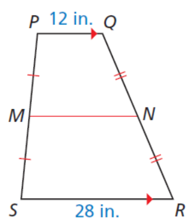
If $\overline{MN}$ is the midsegment of trapezoid $ABCD$, then $\overline{MN} \parallel \overline{AB}$, $\overline{MN} \parallel \overline{DC}$, and $MN = \frac{1}{2}(AB + CD)$.

Proof Ex. 49, p. 406

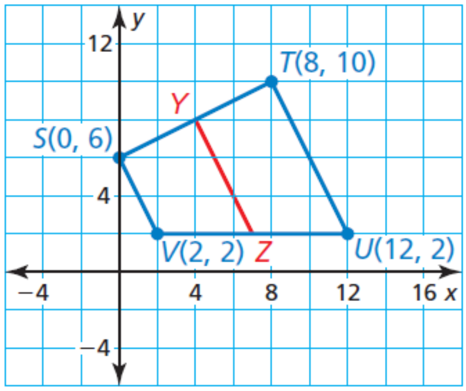


In the diagram, MN is the midsegment of trapezoid PQRS

Find MN.



Find the length of midsegment $\overline{YZ}$ in trapezoid STUV.



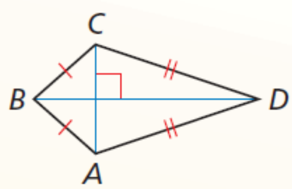
Theorems

Theorem 7.18 Kite Diagonals Theorem

If a quadrilateral is a kite, then its diagonals are perpendicular.

If quadrilateral *ABCD* is a kite, then $\overline{AC} \perp \overline{BD}$.

Proof p. 401

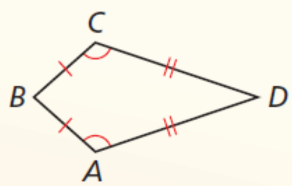


Theorem 7.19 Kite Opposite Angles Theorem

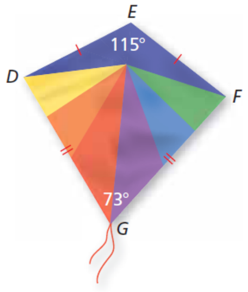
If a quadrilateral is a kite, then exactly one pair of opposite angles are congruent.

If quadrilateral *ABCD* is a kite and $\overline{BC} \cong \overline{BA}$, then $\angle A \cong \angle C$ and $\angle B \ncong \angle D$.

Proof Ex. 47, p. 406

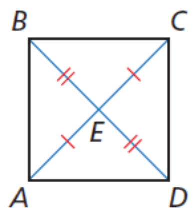


Find m∠D in the kite shown.



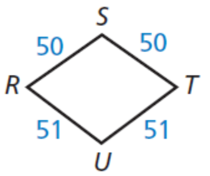
In a kite, the measures of the angles are $3x$°, 75°, 90°, and 120°. Find the value of x. What are the measures of the angles that are congruent?

What is the most specific name for quadrilateral ABCD?

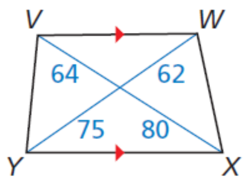


Give the most specific name for the quadrilateral. Explain your reasoning.

8.



9.



10.