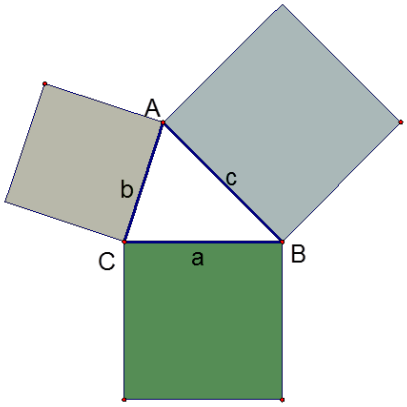
Essential Question

How can you prove the Pythagorean Theorem?



1. $4^2 + 3^2 = x^2$

2. $13^2 + x^2 = 25^2$

3. $\left(\frac{5}{2}\right)^2 + x^2 = \left(\frac{1}{3}\right)^2$

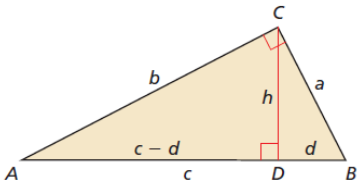
4. $\left(9\sqrt{3}\right)^2 - x^2 = 2^2$

5. $\left(\sqrt{5}\right)^2 + x^2 = 12^2$

6. $\left(5\sqrt{10}\right)^2 - \left(\sqrt{2}\right)^2 = x^2$

Work with a partner.

a. Draw a right triangle with legs a and b, and hypotenuse c, as shown. Draw the altitude from C to $\overline{AB}$. Label the lengths, as shown.



b. Explain why △ABC, △ACD, and △CBD are similar.

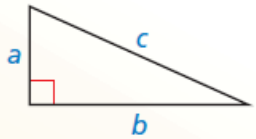
c. Write a two-column proof using the similar triangles in part (b) to prove that $a^2 + b^2 = c^2$.

Theorem

Theorem 9.1 Pythagorean Theorem

In a right triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the legs.

Proof Explorations 1 and 2, p. 463; Ex. 39, p. 484



$c^2 = a^2 + b^2$

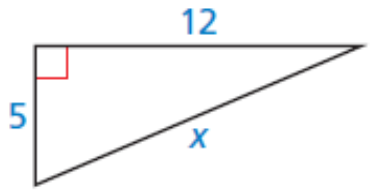
Core Concept

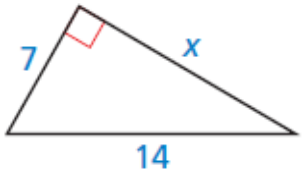
Common Pythagorean Triples and Some of Their Multiples

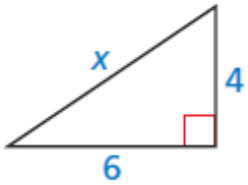
3, 4, 5	**5, 12, 13**	**8, 15, 17**	**7, 24, 25**
6, 8, 10	10, 24, 26	16, 30, 34	14, 48, 50
9, 12, 15	15, 36, 39	24, 45, 51	21, 72, 75
$3x, 4x, 5x$	$5x, 12x, 13x$	$8x, 15x, 17x$	$7x, 24x, 25x$

The most common Pythagorean triples are in bold. The other triples are the result of multiplying each integer in a bold-faced triple by the same factor.

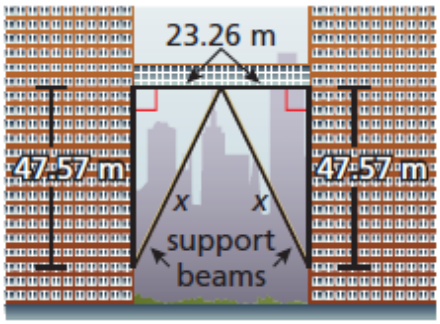
Find the value of x. Then tell whether the side lengths form a Pythagorean triple.







The skyscrapers shown are connected by a skywalk with support beams. Use the Pythagorean Theorem to approximate the length of each support beam.

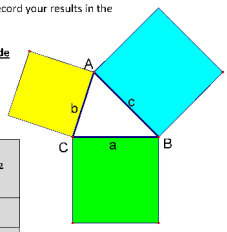


Exploration

Exploring the Pythagorean Theorem With G.S.P.

Use the Geometer's Sketchpad drawing to complete the table below for at least 6 different triangles (2 acute, 2 right, and 2 obtuse). Follow these steps to make a new triangle then record your results in the table:

1. Move *A*, *B*, or *C* to make a new triangle with **c as the longest side length.**
2. Record the type of triangle and the value of a^2, b^2, and c^2 in the table below.



Type of Triangle *Classify by Angles*	a^2	b^2	c^2	$a^2 + b^2$

Converse of the Pythagorean Theorem:

Assume that a, b, and c are the three sides of a triangle and c is the largest side.

If _____, then the triangle is a right triangle

If _____, then the triangle is an acute triangle.

If _____, then the triangle is an obtuse triangle.

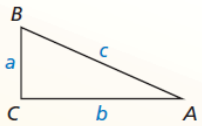
Try These

Textbook Reference: Lesson 7-2

Theorem

Theorem 9.2 Converse of the Pythagorean Theorem

If the square of the length of the longest side of a triangle is equal to the sum of the squares of the lengths of the other two sides, then the triangle is a right triangle.



If $c^2 = a^2 + b^2$, then $\triangle ABC$ is a right triangle.

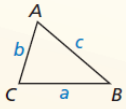
Proof Ex. 39, p. 470

Theorem

Theorem 9.3 Pythagorean Inequalities Theorem

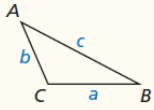
For any $\triangle ABC$, where c is the length of the longest side, the following statements are true.

If $c^2 < a^2 + b^2$, then $\triangle ABC$ is acute.



$c^2 < a^2 + b^2$

If $c^2 > a^2 + b^2$, then $\triangle ABC$ is obtuse.

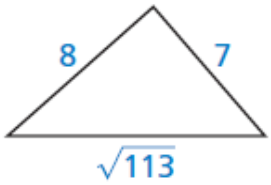


$c^2 > a^2 + b^2$

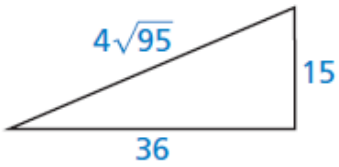
Proof Exs. 42 and 43, p. 470

Tell whether each triangle is a right triangle.

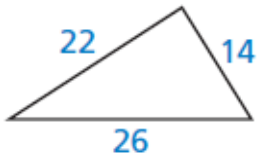
a.



b.



c.



Verify that segments with lengths of 3, 4, and 6 form a triangle. Is the triangle acute, right, or obtuse?