Pre-Calculus Name: Date: Period:

1st Sem. Final Review

Final Exam (RCC Math 111 final)
15 questions – *not* multiple choice
Graphing Calculator required
90 minutes

Unit 1:

1) The following table shows the sales of two Products in millions in the United States for year x, where x is the number of years after 2000.

Year after 2000	0	1	2	3	4
Product A	630	605	583	560	530
Product B	105	162	202	252	309

a) Find the average rate of change in both products between 2000 and 2004. Include units on your answer.

Product A: $\frac{\Delta y}{\Delta x} = \frac{530-630}{4-0} = \frac{-100}{4} = -25;$ *Product B:* $\frac{\Delta y}{\Delta x} = \frac{309-105}{4-0} = \frac{204}{4} = 51$

b) Use the data points for the years 2000 and 2004 to write linear equations to model the growth of Product A and Product B. (Unit 2)

Product A: $y = -25x + 630$
Product B: $y = 51x + 105$

2) *Solve the system algebraically.* Use the equations you found in problem 1b to determine what year Product B will have the same sales as Product A. How many of each type will be sold in that year?

Solve the system:

$$y = -25x + 630$$
$$y = 51x + 105$$

$$\left(x = \frac{525}{76}, y = \frac{34755}{76}\right)$$
$$(6.908, 457.303)$$

During 2006, 457.3 *million units of each will be sold.*

3) Consider the function $f(x) = 3x^3 + 2x^2 - 5x + 6$. Use your calculator to find the following:

a. All extrema

$$(-1, 10), (.556, 4.354)$$

b. List increasing and decreasing intervals

$$\textit{Increasing}: (-\infty, -1) \cup (.556, \infty); \textit{Decreasing}: (-1, .556)$$

c. Real Zeros

$$x = -2, \quad (-2, 0)$$

d. Now use synthetic division to factor.

-2	3	2	-5	6
		-6	8	-6
	3	-4	3	0

$$f(x) = 3x^3 + 2x^2 - 5x + 6 = (x+2)(3x^2 - 4x + 3)$$

Use Quadratic formula:

$$x = \frac{4 \pm \sqrt{(4)^2 - 4(3)(3)}}{2(3)} = \frac{4 \pm \sqrt{-20}}{6} = \frac{4 \pm 2i\sqrt{5}}{6} = \frac{2 \pm i\sqrt{5}}{3}$$

Completely factored:

$$f(x) = 3x^3 + 2x^2 - 5x + 6$$
$$= (x+2)(3x^2 - 4x + 3)$$
$$= (x+2)\left(x - \frac{2 + i\sqrt{5}}{3}\right)\left(x - \frac{2 - i\sqrt{5}}{3}\right)$$

e. Find any imaginary zeros of $f(x)$.

$$x = \frac{2 \pm i\sqrt{5}}{3}$$

4) Consider the function $f(x) = x^2 + x$

a. Find the average rate of change on the interval (1,2).

$$\frac{\Delta y}{\Delta x} = \frac{f(b) - f(a)}{a - b} = \frac{\left((2)^2 + (2)\right) - \left((1)^2 + (1)\right)}{2 - 1} = 4$$

b. Find the difference quotient.

$$D = \frac{f(x+h) - f(x)}{h} = \frac{\left((x+h)^2 + (x+h)\right) - (x^2 + x)}{h} = \frac{2hx + h^2 + h}{h} = 2x + h + 1$$

Unit 2

5) The function $h(t) = -16t^2 + 32t + 256$ models the height of a projectile at time t.

a. At what time does it reach its maximum height?

$$t = \frac{-b}{2a} = -\frac{32}{2(-16)} = 1\ sec$$

b. What is its maximum height?

$$h(1) = -16(1) + 32(1) + 256 = 272$$

c. At what time will the projectile hit the ground?

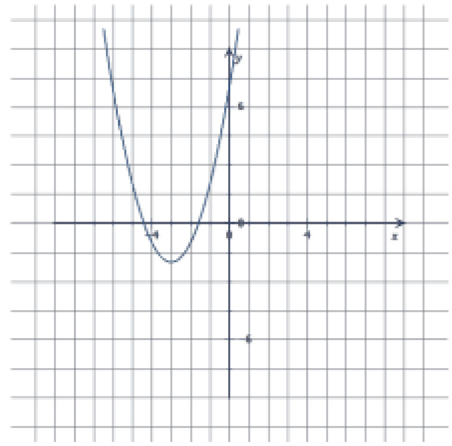
$$0 = -16t^2 + 32t + 256$$
$$t = 1 - \sqrt{17} \text{ or } t = \sqrt{17} + 1$$
$$\textit{Disregard Neg. Answer.}\ t \approx 5.12\ sec.$$

6) The function $f(x) = -.25x^2 + 300x - 25$ represents the sales of a product, $f(x)$, in thousands, x years after 2000. According to this function, in what year will the sales reach a maximum? What will this maximum be?

$$x = -\frac{b}{2a} = -\frac{300}{2(-.25)} = 600$$
$$f(600) = -.25(600)^2 + 300(600) - 25 = 89975$$

7) Complete the square to write the equation in vertex form, identify the coordinates of the vertex, and graph

$$y = x^2 + 6x + 7$$
$$y - 7 = x^2 + 6x$$
$$y - 7 + 9 = x^2 + 6x + 9$$
$$y + 2 = (x+3)^2$$
$$y = (x+3)^2 - 2$$
$$Vertex: (-3, -2)$$



Unit 3

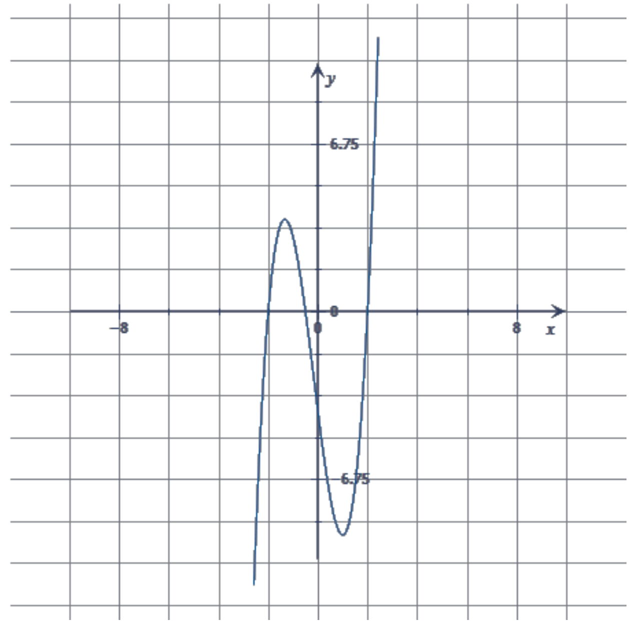
8) Find all zeros (real and imaginary) for $f(x) = x^4 + 3x^3 - 2x^2 - 10x - 12$.

$$From\ Calculator\ (Graphing): x = 2\ or\ x = -3$$
$$After\ Synthetic\ Division: f(x) = (x-2)(x+3)(x^2 + 2x + 2)$$
$$Use\ Quadratic\ formula:$$
$$x = \frac{-2 \pm \sqrt{(2)^2 - 4(1)(2)}}{2(1)} = \frac{-2 \pm 2i}{2} = -1 \pm i$$
$$Zeros: 2, -3, -1+i, -1-i$$

9) Solve the equation $2x^3 + 3x^2 + 7x + 3 = 0$.
You may use your calculator (to start the process), synthetic division, and factoring or the quadratic formula to find all the solutions to the equation. Leave answers as exact values in simplified form.

$$From\ Calculator: x = -\frac{1}{2}$$
$$After\ Synthetic\ Division: \left(x + \frac{1}{2}\right)(2x^2 + 2x + 6) = 0$$
$$\text{or } \left(x + \frac{1}{2}\right)(x^2 + x + 3)2 = \left(x + \frac{1}{2}\right)(x^2 + x + 3) = 0$$
$$Use\ Quadratic\ Formula\ for\ 2nd\ factor:$$
$$x = \frac{-1 \pm \sqrt{1^2 - 4(1)(3)}}{2(1)} = \frac{-1 \pm \sqrt{-11}}{2} = \frac{-1 \pm i\sqrt{11}}{2}$$
$$Solutions: x = \left\{-\frac{1}{2}, \frac{-1 \pm i\sqrt{11}}{2}\right\}$$

10) Write the function $f(x) = 2x^3 + x^2 - 8x - 4$ in completely factored form using your calculator, synthetic division, and/or factoring.



$f(x)$=______________________

$$f(x) = (x-2)(x+2)(2x+1)$$

$$X - intercepts: (2,0)(-2,0)\left(-\frac{1}{2}, 0\right)$$

$$y - intercepts: (0,-4)$$

Sketch a complete graph of the function f(x). Label the coordinates of the x-intercepts and the y-intercept.

11) Consider: $f(x) = \frac{2x^2 - x - 15}{x^2 - 8x + 15}$

a. Write the numerator and denominator in factored form.

$$f(x) = \frac{2x^2 - x - 15}{x^2 - 8x + 15} = \frac{(x-3)(2x+5)}{(x-5)(x-3)}$$

b. State the domain of this function:

$$D: (-\infty, 3) \cup (3,5) \cup (5, \infty)$$

c. Determine the roots (or zeros) of the function and write them as ordered pairs.

$$2x^2 - x - 15 = 0$$

$$x = -\frac{5}{2} \; or \; x = 3$$

$$Since \; x \neq 3, the \; only \; root \; is x = -\frac{5}{2}$$

d. Determine the vertical asymptote(s) and write the equations of the asymptote(s).

$$Vertical \; Asymptote: x = 5$$

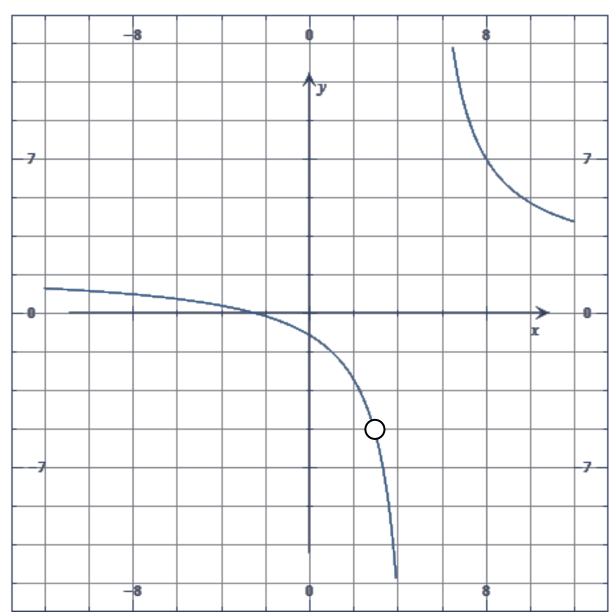
e. Determine any horizontal asymptote and write the equation of the asymptote.

From Long hand division:

$$f(x) = \frac{(2x+5)}{(x-5)} = 2 + \frac{-5}{x-5}$$

$$Horizontal \; Asymptote: y = 2$$

f. Graph the function, its roots, and its asymptotes. Be sure to indicate any holes on the graph. Label your graph.



Unit 4

12) Solve the exponential and logarithmic equations algebraically (You must show your steps to get full credit). Round your answers to three decimal places if necessary.

a) $4(15^x) = 200$

$x = \log_{15} 50$
≈ 1.445

b) $4 + \log_5 \sqrt{2x+3} = 7$

$\sqrt{2x+3} = 5^{7-4}$
$x = 7811$

13) Find the limit of growth and the y-intercept of the logistic function

$$y = \frac{20}{1 + 3(.25)^x}$$

$\textit{Limit of growth}: 20$
$y - \textit{intercept}: (0, 5)$

14) Calculate the number of years necessary for \$250 to grow to \$1000 at 4.5% compounded continuously. Use the compound interest formula: A=Pe^rt, where A = final amount, P = starting amount, r = interest rate, and t = time in years. Show your work and round your answer to the nearest tenth.

$$1000 = 250e^{.045t}$$
$$4 = e^{.045t}$$
$$\ln(4) = .045t$$
$$\frac{\ln(4)}{.045} \approx 30.807$$

15) Consider the following function: $f(x) = 37 + 13\ln(x+9)$. Answer the following to two decimal places.

a) Determine the domain of the function.

$D: (-9, \infty)$

b) Solve for $f(x)$ if $x = 3$

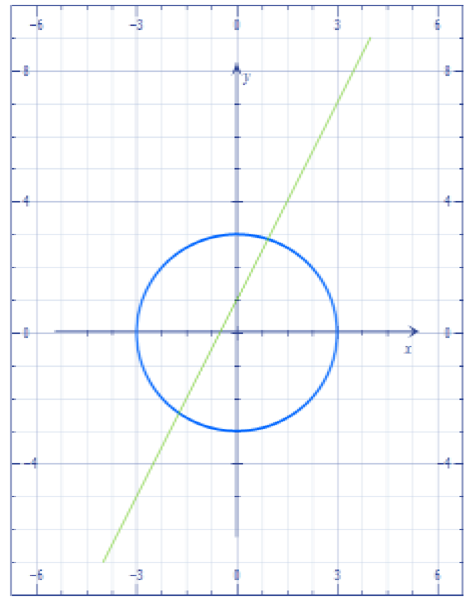
$$f(3) = 37 + 13\,ln(3+9) \approx 69.304$$

c) What value of x gives us $f(x) = 37$?

$$37 = 37 + 13\,ln(x+9)$$
$$x = -8$$

d) What value of x gives us $f(x) = 100$?

$$100 = 37 + 13\ ln(x+9)$$
$$63 = 13\ ln(x+9)$$
$$e^{\frac{63}{13}} = x + 9$$
$$x = e^{\frac{63}{13}} - 9$$



Unit 5

16) Solve the following system of equations algebraically. Then check your answer graphically. Label your solution(s) on the graph.

$$x^2 + y^2 = 9$$
$$y = 2x + 1$$

$$\left(x = \frac{-2\sqrt{11} - 2}{5}, y = \frac{1 - 4\sqrt{11}}{5}\right)\ or$$
$$\left(x = \frac{2\sqrt{11} - 2}{5}, y = \frac{4\sqrt{11} + 1}{5}\right)$$

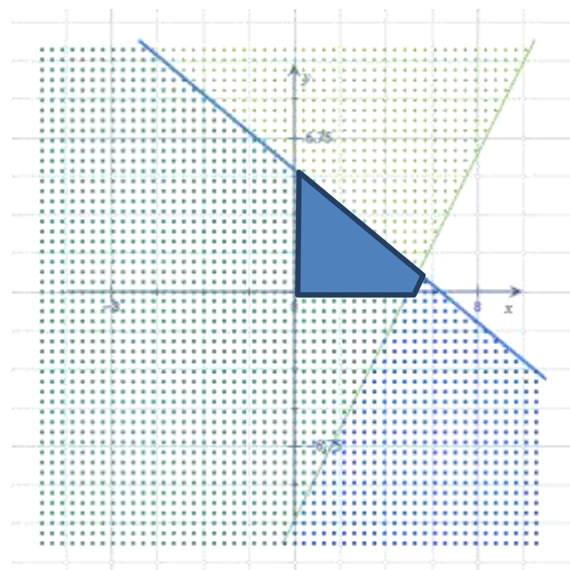
17) Solve the following system of equations using matrices and your calculator. Show the augmented matrix for the system and its reduced row echelon form. Be sure to write out the solution to the system.

$x + y - 2z = -3$
$3x + y + z = 8$
$4x - 3y - z = -5$

$$\left[\begin{array}{ccc|c} 1 & 1 & -2 & -3 \\ 3 & 1 & 1 & 8 \\ 4 & -3 & -1 & -5 \end{array}\right] = \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

$(x = 1, y = 2, z = 3)$

18) Find the maximum value of the objective function, $z = 5x + 3y$, subject to these constraints:



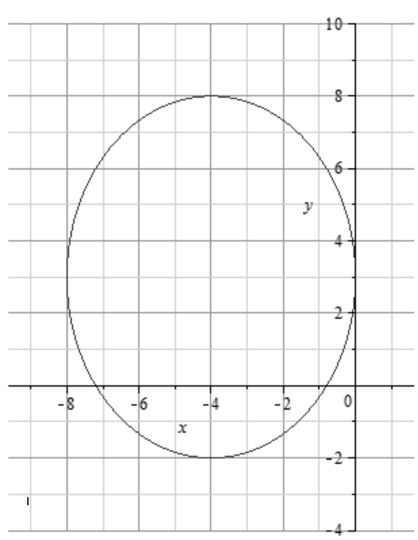
$x \geq 0$
$y \geq 0$
$5x + 6y \leq 32$
$2x - y \leq 10$

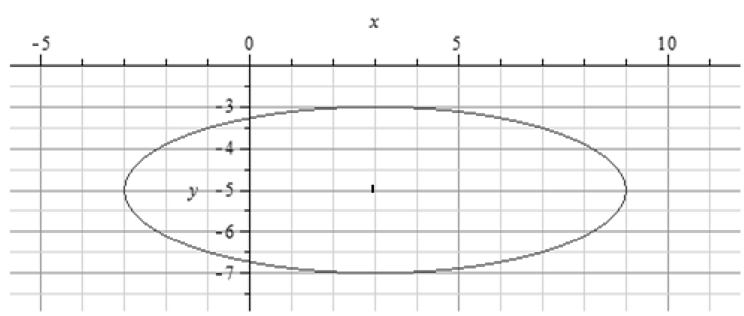
$\boldsymbol{Vertices}: (0,0), \left(\frac{16}{3}, 0\right), (5,0), \left(\frac{92}{17}, \frac{14}{17}\right)$

$Max\ at \left(\frac{92}{17}, \frac{14}{17}\right) \quad 5\left(\frac{92}{17}\right) + 3\left(\frac{14}{17}\right) = \frac{502}{17} = 29.529$

Unit 6

19) Find the equation for the each ellipse below in standard form and calculate the focal distance.





$\frac{(x+4)^2}{16} + \frac{(y-3)^2}{25} = 1$

$\frac{(x-3)^2}{36} + \frac{(y+5)^2}{16} = 1$

20) Change each equation of the circle below to standard form <u>and</u> graph it:

$$x^2 + 3 = -(y-2)^2 + 7$$
$$x^2 + (y-2)^2 = 4$$

$$x^2 + y^2 + 12x - 8y + 43 = 0$$

$$x^2 + 12x + y^2 - 8y = -43$$
$$(x^2 + 12x + 36) + (y^2 - 8y + 16) = -43 + 36 + 16$$
$$(x+6)^2 + (y-4)^2 = 9$$