Pre-Calculus Name: Date: Period:

2nd Semester Final Review 2013 **Selected SOLUTIONS**

(some duplicates are left for you to solve.)

Work through each of the problems on a separate piece of paper so that you can use this form to practice the problems again. This assignment will not be collected – this is your opportunity to begin preparing and studying for the final. Working with other students in class and forming study groups to go over these problems is highly recommended.

The formula sheet that is given out with this review is the same one that you will be allowed to use on the final. It will serve as your only notes on the final. You will be given a new, clean, copy of the formula sheet during the final.

The final will be given over two days on Thursday and Friday, May 30th and 31st. You will one class period to complete 7 or 8 problems each day.

1) A circle has a radius of 10".

a. It is cut into seven equal pieces. What angle, in degrees, AND convert to radians, will be formed by each piece?

$$a = \left(\frac{360}{7}\right)^\circ = 51\frac{3}{7}^\circ$$
$$a = \frac{360^\circ}{7} \cdot \frac{\pi}{180^\circ} = \frac{2\pi}{7}\ radian$$

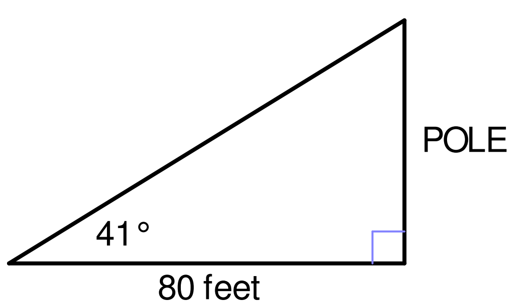
b. If the circle is spun in a complete rotation (that is 2π radians) every ¼ second, what is the speed of the outer edge of the circle in inches per second?

$$r = \frac{L}{t} = \frac{2\pi(10)in.}{\frac{1}{4}sec} = 80\pi\ in/sec$$

2) A bike tire has a radius of 22". If the tire spins 200 times in 60 seconds, how fast is the bike moving?

$$r = \frac{L}{t} = \frac{200(2\pi(22))\ in}{60\ sec} = \frac{440\pi}{3} = 460.76\ in/sec$$

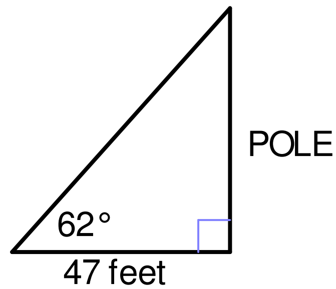
3) The angle of elevation to the top of pole is 41°. If the distance to the pole is 80 feet, what is the height of the pole?



$\tan 41 = \frac{x}{80}$

$x = 80 \tan 41 = 69.54\ ft.$

4) The angle of elevation to the top of pole is 62°. If the distance to the pole is 47 feet, what is the height of the pole?



$$\tan 62 = \frac{x}{47}$$

$$x = 47 \tan 62 = \mathbf{88.39ft.}$$

5) Find the exact value of the following expression without using your calculator. Show your diagram for full credit.

$\cos\left(\sin^{-1}\left(\frac{3}{7}\right)\right)$

$$\theta = \sin^{-1}\left(\frac{3}{7}\right) \to opp = 3, hyp = 7 \to adj = \sqrt{49-9} = \sqrt{40} = 2\sqrt{10}$$

$$\cos(\theta) = \frac{2\sqrt{10}}{7}$$

6) Find the exact value of the following expression without using your calculator. Show your diagram for full credit.

$\sin\left(\tan^{-1}\left(\frac{8}{5}\right)\right)$

$$\theta = \tan^{-1}\left(\frac{8}{5}\right) \to opp = 8\,, adj = 5 \to hyp = \sqrt{89}$$

$$\sin\theta = \frac{8}{\sqrt{89}}$$

7) Simplify the expression $\frac{\csc x \cdot \tan^2 x}{\sec x \cdot \sin^2 x + \cos x}$

$$\frac{\csc x \cdot \tan^2 x}{\sec x \cdot \sin^2 x + \cos x} = \frac{\frac{1}{\sin x} \cdot \frac{\sin^2 x}{\cos^2 x}}{\frac{1}{\cos x}\sin^2 x + \cos x} \cdot \frac{\cos^2 x}{\cos^2 x} = \frac{\sin x}{\cos x \sin^2 x + \cos^3 x}$$

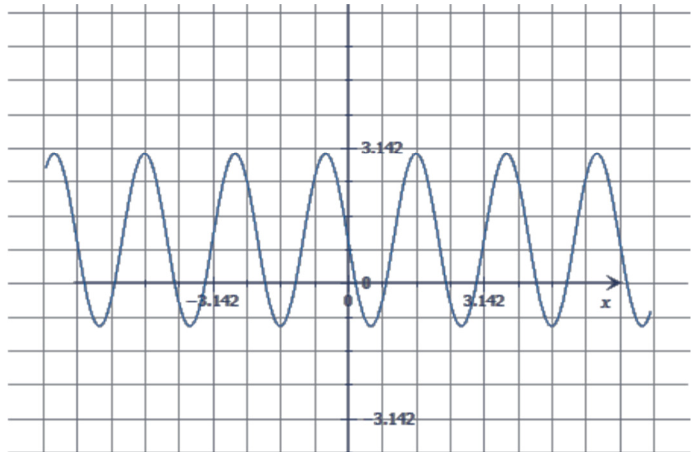
$$= \frac{\sin x}{\cos x\,(\sin^2 x + \cos^2 x)} = \frac{\sin x}{\cos x} = \tan x$$

8) List the amplitude, period, phase shift and vertical shift for the trig function:

$$y = 2\cos\left(-3\left(x - \frac{\pi}{2}\right)\right) + 1$$

Amp=2, Per=$-\frac{2\pi}{3}$, Phase Shift=$\frac{\pi}{2}$, vertical shift=1

Use a grid like the one below to graph the function on the grid below. Be sure to clearly label key points.



9) Solve for x in the equation on the interval $[0, 2\pi)$. Solve algebraically and give exact values for the answers.

$$\tan x - \frac{\sqrt{3}}{3} = 0$$

$$\tan x = \frac{\sqrt{3}}{3} \to opp = \sqrt{3}, adj = 3 \to hyp = 2\sqrt{3}$$

$$\cos x = \pm\frac{3}{2\sqrt{3}} = \pm\frac{\sqrt{3}}{2}, \quad \text{and } \sin x = \pm\frac{\sqrt{3}}{2\sqrt{3}} = \pm\frac{1}{2}$$

Note: since tangent is positive and $\tan x = \frac{\sin x}{\cos x}$, both coordinates must have the same sign.

So, the unit circle points are $\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right), \left(\frac{-\sqrt{3}}{2}, \frac{-1}{2}\right)$ giving us the angles

$$x = \frac{\pi}{6}, \frac{7\pi}{6}$$

10) Solve for x in the equation on the interval $[0, 2\pi)$. Solve algebraically and give exact values for the answers.

$$\sin x + \frac{1}{2} = 0$$

$$\sin x = -\frac{1}{2}$$

$$x = \frac{7\pi}{6}, \frac{11\pi}{6}$$

11) Find <u>all</u> of the solutions to the following equation. Solve algebraically and give exact values for the answers.

$$-2\cos^2 x + 3\sin x + 3 = 0$$

Substitute $\cos^2 x = 1 - \sin^2 x$

$$-2(1 - \sin^2 x) + 3\sin x + 3 = 0$$

$$2\sin^2 x + 3\sin x + 1 = 0$$

$$(2\sin x + 1)(\sin x + 1) = 0$$

$$2\sin x + 1 = 0, \quad \text{or } \sin x + 1 = 0$$

$$\sin x = -\frac{1}{2}, \quad \text{or } \sin x = -1$$

$$x = \frac{7\pi}{6}, \frac{11\pi}{6}, \frac{3\pi}{2}$$

12) Find the **exact value** of the expression below.

Given that $\tan\alpha = -\frac{4}{3}$ for α in Quadrant IV and $\cos\beta = -\frac{5}{13}$ for β in Quadrant II, find $\cos(\alpha - \beta)$.

$$\tan\alpha = -\frac{4}{3} \to opp = -4, adj = 3, hyp = 5 \to \sin\alpha = -\frac{4}{5}, \cos\alpha = \frac{3}{5}$$
$$\cos\beta = -\frac{5}{13} \to adj = -5, hyp = 13, opp = 12 \to \sin\beta = \frac{12}{13}, \cos\beta = -\frac{5}{13}$$
$$\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta = \frac{3}{5}\left(-\frac{5}{13}\right) + \left(-\frac{4}{5}\right)\left(\frac{12}{13}\right) = -\frac{63}{65}$$

13) Find the **exact value** of the expression below.

Given that $\cos\alpha = \frac{12}{13}$ for α in Quadrant IV and $\tan\beta = \frac{3}{4}$ for β in Quadrant I find $\cos(\alpha - \beta)$.

14) Solve the triangle. Round to the nearest tenth and nearest degree for sides and angles, respectively.

$$a = 13, \quad c = 7.9, \quad A = 51°$$
$$\frac{\sin 51}{13} = \frac{\sin C}{7.9}$$
$$C = \sin^{-1}\left(7.9 \cdot \frac{\sin 51}{13}\right) \approx 28.1815° \approx 28°$$
$$B = 180 - (51 + 28.1815) \approx 100.8185 \approx 101°$$
$$\frac{\sin 100.8185}{b} = \frac{\sin 51}{13}$$
$$b = \frac{13 \sin 100.8185}{\sin 51} \approx 16.4306 \approx 16.4$$

We consider if there are two possibilities for this triangle. This is true if

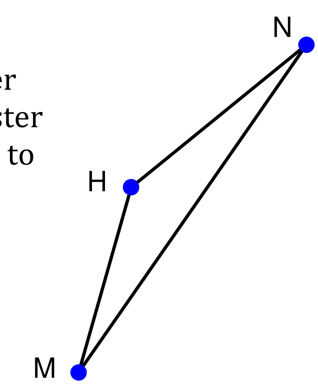
$$C \approx 180 - 28.1815 \approx 151.8185° \approx 152°$$

But this is not possible since $152° + 51° > 180$. So, there is only one triangle possible.

15) Solve the triangle. Round to the nearest tenth and nearest degree for sides and angles, respectively.

$$a = 14.2, \; c = 9.5, \; A = 73°$$

16) Two sisters leave their house (point H) at the same time. One sister travels on a bearing of 195° at 2.3 mph to location M. The other sister travels on a bearing of 42° at 2.8 mph to location N. How far apart, to the nearest mile, will the sisters be after **three** hours?



The angle between $\overline{HN}$ and $\overline{HM}$ is

$$m\angle H = 195 - 42 = 153°$$

The distance walked by each person is

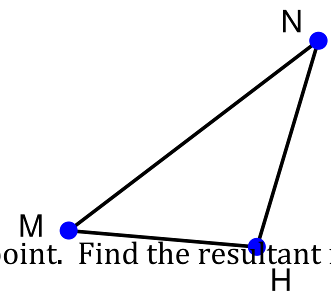
$$3 \cdot 2.3 = 6.9 \text{ mi., and } 3 \cdot 2.8 = 8.4 \text{ mi.}$$

Using the law of Cosines

$$x^2 = 6.9^2 + 8.4^2 - 2(6.9)(8.4)(\cos 153)$$

Distance: $x \approx 14.88 \approx$ **15 mi.**

17) Two sisters leave their house (point H) at the same time. One sister travels on a bearing of 280° at 2.1 mph to location M. The other sister travels on a bearing of 20° at 2.3 mph to location N. How far apart, to the nearest mile, will the sisters be after **three** hours?



18) Consider force vectors **u** & **v** acting on the same point. Find the resultant magnitude and angle θ_R.

$\|u\| = 220$ pounds, $\theta = 29°$

$\|v\| = 300$ pounds, $\theta = 205°$

$$u = \langle 220 \cos 29, 220 \sin 29 \rangle$$
$$v = \langle 300 \cos 205, 300 \sin 205 \rangle$$
$$r = u + v = \langle 220 \cos 29 + 300 \cos 205, 220 \sin 29 + 300 \sin 205 \rangle$$
$$r = \langle -79.476, -20.127 \rangle$$
$$\sqrt{(-79.476)^2 + (-20.127)^2}$$
$$\|r\| = \sqrt{(-79.476)^2 + (-20.127)^2} \approx 81.985 \text{ pounds}$$
$$\tan^{-1} \frac{-20.127}{-79.476} \approx 14.21°$$
$$\theta_r = 180 + 14.21 = 194.21°$$

19) Consider force vectors **u** & **v** acting on the same point. Find the resultant magnitude and angle θ_R.

$\|u\| = 100$ pounds, $\theta = -52°$

$\|v\| = 254$ pounds, $\theta = 325$

20) A force of 75 pounds on a rope is used to pull a box up a ramp inclined at 12° from the horizontal. The rope forms an angle of 40° with the horizontal. How much work is done pulling the box 22 feet along the ramp?

We know that $W = \|\vec{F}\| \cdot \|\vec{AB}\| \cos \theta$

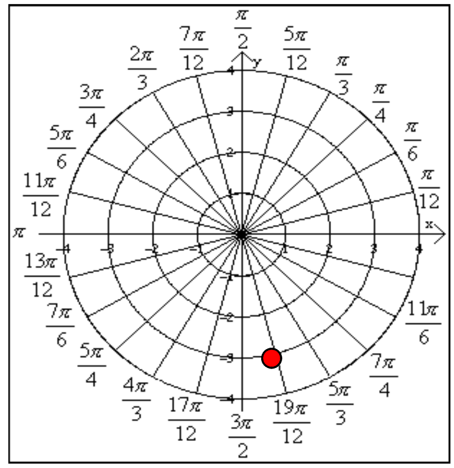
$$\|\vec{F}\| = 75, \|\vec{AB}\| = 22$$
$$\theta = 40 - 12 = 28°$$
$$W = 75 \cdot 22 \cdot \cos 28 \approx 1456.86 \text{ foot} - \text{pounds}$$

21) A force of 40 pounds on a rope is used to pull a box up a ramp inclined at 20° from the horizontal. The rope forms an angle of 35° with the horizontal. How much work is done pulling the box 50 feet along the ramp?

22) Given the point (r, θ) $= \left(-3, \frac{7\pi}{12}\right)$:

a) Plot the point in the polar coordinate system.

See Graph



b) Give another representation of the point in polar form using a positive r value.

$\left(3, \frac{19\pi}{12}\right)$

c) Convert the point to rectangular coordinates (x,y). (Leave as exact values)

$$x = 3\cos\frac{19\pi}{12} = 3\cos(-75°) = 3\cos 75° = 3\cos(45+30)$$

$$= 3(\cos 45\cos 30 - \sin 45 \sin 30) = 3\left(\frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2}\cdot\frac{1}{2}\right) = \frac{3\sqrt{6} - 3\sqrt{2}}{4}$$

$$y = 3\sin\frac{19\pi}{12} = 3\sin(-75) = -3\sin(75) = -3\sin(45+30)$$

$$= -3(\sin 45\cos 30 + \sin 30\cos 45 = -3\left(\frac{\sqrt{2}}{2}\cdot\frac{1}{2} + \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2}\right)$$

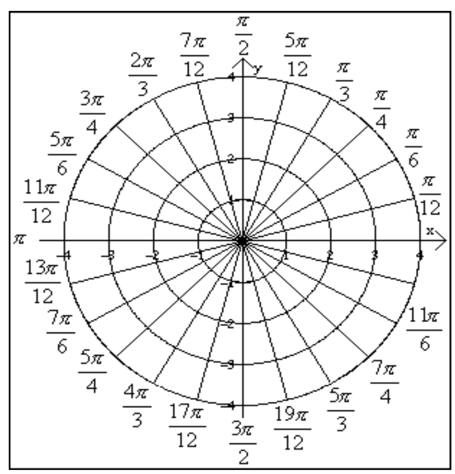
$$= \frac{-3\sqrt{2} - 3\sqrt{6}}{4}$$

Cartesian Coordinates:

$$\left(\frac{3\sqrt{6} - 3\sqrt{2}}{4}, \frac{-3\sqrt{2} - 3\sqrt{6}}{4}\right)$$

23) Given the point (r, θ) $= \left(2, \frac{\pi}{4}\right)$:

a) Plot the point in the polar coordinate system.



b) Give another representation of the point in polar form using a negative r value.

c) Convert the point to rectangular coordinates (x,y). (Leave as exact values)