

5A: Systems of Equations of 2 Variables

In past Algebra classes, you have learned how to solve systems of *linear* equations using several methods. Now we will see how we can use the same methods with *non-linear* methods.

Warmup: Car loans

You just graduated from college and you just got hired by a law firm that is going to purchase you a new Ford Mustang convertible for \$27200! You are going to get a 60 month loan (*even though you may not keep it that long*) and you have two options:

Option 1: \$2000 down payment and \$475 per month

Option 2: \$0 down payment and \$525 per month



- For each option, write an equation to represent the total cost y for keeping the car for x months.
- Graph both equations with your calculator.
- How would this information help you to decide which option to choose?

Systems of Equations

A **system of equations** is a set of equations that are considered simultaneously. That is, their variables represent the same quantities.

A **solution of a system** is the ordered pair (for a 2 variable system) or ordered n -tuple (for a system with n variables) that is a solutions for all equations in the system.

Standard Methods for Solving Systems

Method 1: Graphing

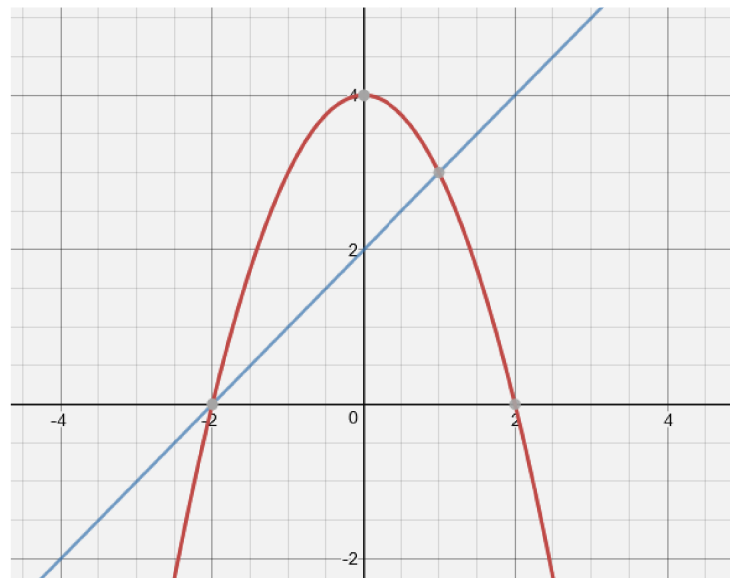
Remember that the graph of an equation is a picture of all the solutions of the equation. If we have two (or more) equations to solve simultaneously, we can graph them to find the point(s) that they intersect, if any.

Note: If there is no intersection of the graphs, then there is *no solution*.

Example: Solve by graphing.

$$y = -x^2 + 4$$

$$y = x + 2$$



Method 2: Substitution

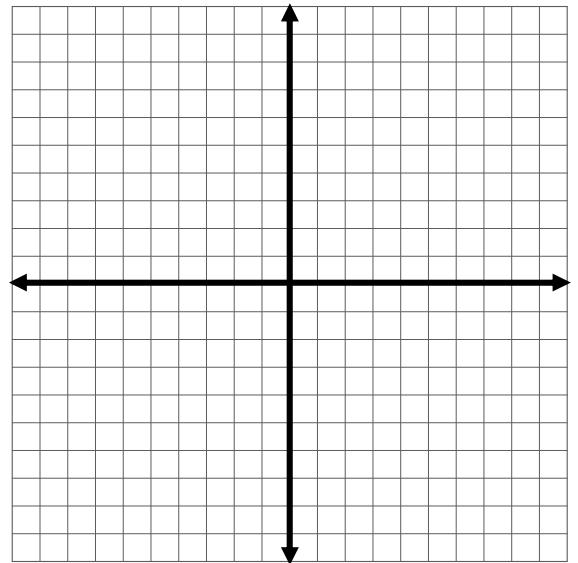
To solve by substitution, solve for one variable and substitute the result into the other equation.

Example: Solve the system in the previous example using substitution

$$y = -x^2 + 4$$

$$y = x + 2$$

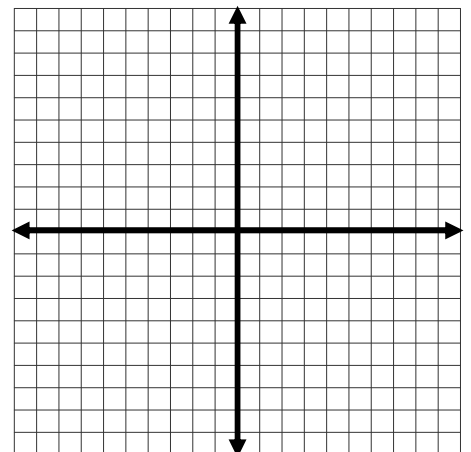
Explore Find all the integers coordinates (x, y) that satisfy the equation $x^2 + y^2 = 5^2$.
(Hint: think about the Pythagorean Theorem.)
Graph these points.
What shape would you get if you find all the non-integer solutions as well?



Example: Solve by substitution and check by graphing

$$x^2 + y^2 = 25$$

$$y = x + 2$$



Method 3: Elimination

To solve by elimination, write both equations in equivalent forms and add or subtract the equations to eliminate one variable.

Example: Solve by elimination

$$3x - 2y = 10$$

$$x + y = 6$$

Example: Solve by elimination

$$y = 2x^2 + 4x + 1$$

$$y = x^2 + 4x + 2$$